Control by Interconnection and Standard Passivity–Based Control of Port–Hamiltonian Systems

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Abstract— The dynamics of many physical processes can be suitably described by Port–Hamiltonian (PH) models, where the importance of the energy function, the interconnection pattern and the dissipation of the system is underscored. To regulate the behavior of PH systems it is natural to adopt a Passivity–Based Control (PBC) perspective, where the control objectives are achieved shaping the energy function and adding dissipation. In this paper we consider the PBC techniques of Control by Interconnection (CbI) and Standard PBC. In CbI the controller is another PH system connected to the plant (through a power–preserving interconnection) to add up their energy functions, while in Standard PBC energy shaping is achieved via static state feedback. In spite of the conceptual appeal of formulating the control problem as the interaction of dynamical systems, the current version of CbI imposes a severe restriction on the plant dissipation structure that stymies its practical application. On the other hand, Standard PBC, which is usually derived from a uninspiring and non–intuitive "passive output generation" viewpoint, is one of the most successful controller design techniques. The main objectives of this paper are: (1) To extend the CbI method to make it more widely applicable—in particular, to overcome the aforementioned dissipation obstacle. (2) To show that various popular variants of Standard PBC can be derived proceeding from a unified perspective. (3) To establish the connections between CbI and Standard PBC proving that the latter is obtained restricting the former to a suitable subset—providing a nice geometric interpretation to Standard PBC—and comparing the size of the set of PH plants for which they are applicable.

Index Terms— Passivity, nonlinear systems, stabilization, interconnection, passivity–based control, Hamiltonian systems.

I. INTRODUCTION

In the last few years we have witnessed in the control literature, both theoretical and applied, an ever increasing predominance of control techniques that respect, and effectively exploit, the structure of the system over the more classical techniques that try to impose some predetermined dynamic behavior—usually through nonlinearity cancellation and high gain. The property of passivity plays a central role in most of these developments. Passivity–Based Control (PBC) is a generic name, introduced in [26], to define a controller design methodology which achieves the control objective, e.g., stabilization, by rendering the system passive with respect to a desired storage function and injecting damping. There are many variations of the basic PBC idea, and we refer the interested reader to [8], [23], [29], [32], [34] for further details and a list of references.

In this paper we are interested in the control of dynamical systems endowed with a special geometric structure, called a Port–Hamiltonian (PH) model. As shown in [33], [34], PH models provide a suitable representation of many physical processes and have the essential feature of underscoring the importance of the energy function, the interconnection pattern and the dissipation of the system.[1] There are many possible representations of PH models, here we will consider the so–called input–state–output form, where the state is assumed finite dimensional and the port variables are the input and output vectors, which satisfy a cyclo–passivity inequality. (The distinction between cyclo–passivity and the more standard passivity property will be discussed later.) To regulate the behavior of PH systems it is natural then to adopt a PBC perspective.

We consider in this paper the PBC techniques of Control by Interconnection (CbI) [6], [24] and Standard PBC [3], [8], [23], [25], [26], [29], [32]. In CbI the controller is another

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[1]Central to the formulation of PH models is the geometric notion of a Dirac structure. We will not elaborate any further on this powerful concept here and refer the reader to [33] for more information.

PH system with its own state variables and energy function. The regulator and the plant are interconnected in a power–preserving way, that is, through a loss–less subsystem. A straightforward application of the passivity theorem [7] shows that the overall system is still cyclo–passive with new energy function the sum of the energy functions of the plant and the controller. To assign to the overall energy function a desired shape, it is necessary to "relate" the states of the plant and the controller via the generation of invariant sets—defined by, so–called, Casimir functions. In its basic formulation, CbI assumes that only the plant output is measurable and considers the classical output feedback interconnection. In this case, the Casimir functions are fully determined by the plant, which imposes a severe restriction on the plant dissipation structure. It has been shown in [24] that, roughly speaking, "dissipation cannot be present on the coordinates to be shaped". This, so–called, dissipation obstacle stymies the use of CbI for applications other than mechanical systems where the coordinates to be shaped are typically positions, which are unaffected by friction.

The first objective of our work is to extend the conceptually appealing CbI method to make it more widely applicable—in particular, to overcome the aforementioned dissipation obstacle. Towards this end, we introduce two extensions to the method. First, exploiting the non–uniqueness of the PH representation of the system, we propose a procedure to generate new cyclo–passive outputs (with new storage functions). Applying CbI through these new port variables overcomes the dissipation obstacle, but still rules out several interesting physical examples—not surprisingly since this is still an output feedback control strategy. Our second, and key modification, assumes that the plant state variables are available for measurement, and proposes to replace the simple output feedback by a suitably defined state–modulated interconnection. In this way, the conditions for existence of Casimir functions can be further relaxed, enlarging the class of PH plants for which the method is applicable.

We also consider in the paper Standard PBC, where energy shaping is achieved via static state feedback and damping is injected feeding back the passive output. Standard PBC, which is usually derived from a uninspiring and non–intuitive "passive output generation" viewpoint, is currently one of the most successful controller design techniques, that includes Energy–Balancing (EB), Interconnection and Damping Assignment (IDA) and Power–Shaping (PS) PBC. A second objective of this paper is to show that all these variants of Standard PBC can be naturally derived in a systematic way: selecting the desired closed–loop dissipation.

The third objective of the paper is to relate and compare CbI and Standard PBC, which is done with three different criteria. First, comparing the size of the set of PH plants for which they are applicable—this is in its turn determined by the size of the solution set of the partial differential equations (PDEs) that need to be solved for each of the methods. Second, proving that the (static feedback) Standard PBC laws are the restriction of the (dynamic feedback) CbI on the invariant sets defined by the Casimir functions. This provides a nice geometric interpretation to this successful controller design

technique.[2] Finally, it is shown that if CbI can stabilize a given plant then this is also possible with the corresponding Standard PBC—proving that, from the stabilization viewpoint, there is no advantage in considering dynamic feedback.

The remaining of the paper is organized as follows. In Section II we review the basic scheme of CbI for PH systems and exhibit the dissipation obstacle. Section III is devoted to the generation of new cyclo–passivity properties for the system and apply CbI to these new cyclo–passive systems in Section IV. The use of state–modulated interconnections in CbI is presented in Section V. The derivation of various Standard PBCs, proceeding from the selection of the desired dissipation, is carried out in Section VI, while the connections between CbI and Standard PBC are established in Section VII. Some illustrative academic examples are presented in Section VIII and we wrap–up the paper with concluding remarks and future research in Section IX. For ease of reference, a list of acronyms (that, alas, plague this paper) is given in the appendix.

Notation All vectors defined in the paper are *column* vectors, even the gradient of a scalar function that we denote with the operator $\nabla_x = \frac{\partial}{\partial x}$. When clear from the context the subindex of the operator ∇ and the arguments of the functions will be omitted. For vector functions $\mathcal{F} : \mathbb{R}^n \to \mathbb{R}^m$, we define its (transposed) Jacobian matrix $\nabla \mathcal{F}(x) \overset{\triangle}{=} [\nabla \mathcal{F}_1(x), \ldots, \nabla \mathcal{F}_m(x)]$ and, for a distinguished element $x_\star \in \mathbb{R}^n$, we denote $\mathcal{F}_\star \overset{\triangle}{=} \mathcal{F}(x_\star)$.

II. CONTROL BY INTERCONNECTION OF PH SYSTEMS

In order to make this paper self–contained, after presenting PH models, we briefly review in this section the basic version of the CbI method, and discuss its limitations in the presence of dissipation.

A. Cyclo–Passivity of Port–Hamiltonian Systems

PH models of power–conserving physical systems were introduced in [21], see [24], [33], [34] for a review. The input–state–output representation of PH systems is of the form

$$\Sigma_{(u,y)} \begin{cases} \dot{x} &= [\mathcal{J}(x) - \mathcal{R}(x)]\nabla H(x) + g(x)u \\ y &= g^\top(x)\nabla H(x), \end{cases} \quad (1)$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$, $m \leq n$, is the control action, $H : \mathbb{R}^n \to \mathbb{R}$ is the total stored energy, $\mathcal{J}, \mathcal{R} : \mathbb{R}^n \to \mathbb{R}^{n \times n}$, with $\mathcal{J} = -\mathcal{J}^\top$ and $\mathcal{R} = \mathcal{R}^\top \geq 0$, are the natural interconnection and damping matrices, respectively, u, $y \in \mathbb{R}^m$, are conjugated variables whose product has units of power and $g : \mathbb{R}^n \to \mathbb{R}^{n \times m}$ is assumed full rank. We bring to the readers attention the important fact that H is not assumed to be positive semi–definite (nor bounded from below). Also, to simplify the notation in the sequel we define the matrix $F : \mathbb{R}^n \to \mathbb{R}^{n \times n}$

$$F(x) \overset{\triangle}{=} \mathcal{J}(x) - \mathcal{R}(x),$$

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[2]At a more fundamental level, viewing Standard PBC as (a restriction of) interconnected subsystems is consistent with the behavioral framework [22], which rightfully claims that the classical input–to–output assignment perspective is unsuitable to deal, at an appropriately general level, with the basic tenets of systems theory.

which clearly satisfies

$$F + F^\top = -2\mathcal{R} \le 0. \tag{2}$$

The power conservation property of PH systems is captured by the power–balance equation

$$\dot{H} = -\left(\nabla H\right)^\top \mathcal{R}\nabla H + u^\top y. \tag{3}$$

Using the fact that $\mathcal{R} \ge 0$ we obtain the bound

$$\dot{H} \le u^\top y, \tag{4}$$

that, following the original denomination of [36], we refer as cyclo–passivity inequality. Systems satisfying such an inequality are called cyclo–passive, which should be distinguished from passive systems where H is positive semi–definite.[3]

Remark 1: In words, a system is cyclo–passive when it cannot create energy over closed paths in the state–space. It might, however, produce energy along some initial portion of such a trajectory; if so, it would not be passive. On the other hand, every passive system is cyclo–passive. It has been shown in [11] that, similarly to passive systems, one can use storage functions and passivity inequalities to characterize cyclo–passivity provided we eliminate the restriction that these storage functions be non–negative.

Remark 2: Although the paper considers only systems described by PH models (1) some of the results are applicable to the more general class of cyclo–passive systems

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= g^\top(x)\nabla V(x), \end{aligned}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ and $V : \mathbb{R}^n \to \mathbb{R}$ satisfy $f^\top \nabla V \le 0$. (This class has been considered, for instance, in [11].) Under which conditions f can be expressed as $F\nabla V$, for some F verifying $F + F^\top \le 0$, is a difficult question. An affirmative (constructive) answer has been given in [27], but the proposed F has singularities. See also [18], [29], [35] and the discussion in Subsection 4.2.2 of [34].

B. Energy Shaping via Control by Interconnection

As indicated above, in PBC the control objective is achieved rendering the system passive with respect to a desired storage function and injecting damping. For the basic problem of stabilization, the desired energy function should have a minimum at the equilibrium and the damping injection insures that the function is non–increasing. In this way, the energy function qualifies as a Lyapunov function. We now briefly review the PBC method of CbI for stabilization of PH systems, we refer the reader to [33], [34] for further details and extensions. The configuration used for CbI is shown in Fig. 1, where the controller, Σ_c, is a PH system, coupled with the plant, $\Sigma_{(u,y)}$, via the interconnection subsystem, Σ_I, that we select to be power–preserving. That is, such that, for all $t \ge 0$,

$$y^\top(t)u(t) + y_c^\top(t)u_c(t) = y^\top(t)v(t), \tag{5}$$

where v is an external signal that we introduce to define the port variables of the interconnected system and (possibly) inject additional damping.

[3]In [24], [27] we referred to cyclo–passive systems as energy–balancing.
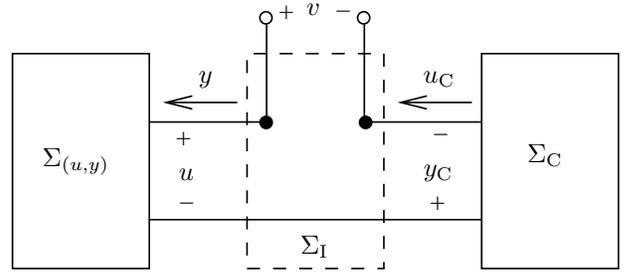
Fig. 1. Block diagram of the CbI scheme with external port variables (v, y).

We choose the dynamics of the controller to be a simple set of (possibly nonlinear) integrators, that is,

$$\Sigma_c : \begin{cases} \dot{\zeta} &= u_c \\ y_c &= \nabla H_c(\zeta), \end{cases} \tag{6}$$

where $\zeta, u_c, y_c \in \mathbb{R}^m$, and $H_c : \mathbb{R}^m \to \mathbb{R}$ is the controllers energy function—to be defined by the designer. From

$$\dot{H}_c = u_c^\top y_c, \tag{7}$$

we see that Σ_c is cyclo–passive (actually, cyclo–lossless). In its simplest formulation, CbI assumes that we measure only the plant output and fixes Σ_I to be the standard negative feedback interconnection

$$\Sigma_I : \begin{cases} \begin{bmatrix} u \\ u_c \end{bmatrix} = \begin{bmatrix} 0 & -I_m \\ I_m & 0 \end{bmatrix} \begin{bmatrix} y \\ y_c \end{bmatrix} + \begin{bmatrix} v \\ 0 \end{bmatrix}, \end{cases} \tag{8}$$

which clearly satisfies (5), with I_m the $m \times m$ unitary matrix. Combining (4), (5) and (7), we obtain that the interconnected system is also cyclo–passive with port variables (v, y) and energy function the sum of the energy functions of the plant and the controller, that is

$$\dot{H} + \dot{H}_c \le v^\top y. \tag{9}$$

To complete the shaping of the energy function CbI invokes the Energy–Casimir method—well–known in Hamiltonian systems analysis, see e.g. [6], [19]—and looks for conserved quantities (dynamical invariants) of the overall system. If such quantities can be found we can generate Lyapunov function candidates combining the conserved quantities and the energy function. We will look, in particular, for conserved quantities that are *independent* of the energy functions H and H_c—such functions are called Casimir.

The application of the Energy–Casimir method for stability analysis of (output feedback) CbI is summarized below.

Proposition 1: Consider the PH system $\Sigma_{(u,y)}$ (1) coupled with the PH controller Σ_c (6) through the power–preserving interconnection subsystem Σ_I (8). Assume there exists a vector function $\mathcal{C} : \mathbb{R}^n \to \mathbb{R}^m$ such that

$$\begin{bmatrix} F^\top \\ g^\top \end{bmatrix} \nabla \mathcal{C} = \begin{bmatrix} g \\ 0 \end{bmatrix} \tag{10}$$

Then, for all functions $\Phi : \mathbb{R}^m \to \mathbb{R}$, the function $W : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$

$$W(x, \zeta) \triangleq H(x) + H_c(\zeta) + \Phi(\mathcal{C}(x) - \zeta), \tag{11}$$

is such that

$$\dot{W} \leq v^\top y. \tag{12}$$

Hence, the system is cyclo–passive with storage function W.

Proof: The dynamics of the interconnected system is given by

$$\begin{bmatrix} \dot{x} \\ \dot{\zeta} \end{bmatrix} = \begin{bmatrix} F & -g \\ g^\top & 0 \end{bmatrix} \begin{bmatrix} \nabla H \\ \nabla H_c \end{bmatrix} + \begin{bmatrix} gv \\ 0 \end{bmatrix}.$$

Now,

$$\dot{\mathcal{C}} - \dot{\zeta} = \begin{bmatrix} (\nabla \mathcal{C})^\top & | & -I_m \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\zeta} \end{bmatrix}.$$

Evaluating along the closed–loop dynamics above and invoking (10), yields $\dot{\mathcal{C}} - \dot{\zeta} = 0$, for all H and H_c. Hence, $\dot{\Phi} = 0$. This, together with (9) and (11) completes the proof. ∎

Remark 3: For ease of notation, and with some loss of generality, we have taken the order of Σ_c to be equal to the number of inputs. If we let $\zeta \in \mathbb{R}^r$, for any $r \in \mathbb{Z}_+$, we should replace the interconnection subsystem (8) by

$$\begin{bmatrix} u \\ u_c \end{bmatrix} = \begin{bmatrix} 0 & -\alpha \\ \alpha^\top & 0 \end{bmatrix} \begin{bmatrix} y \\ y_c \end{bmatrix} + \begin{bmatrix} v \\ 0 \end{bmatrix},$$

where $\alpha \in \mathbb{R}^{m \times r}$. All derivations in this section, i.e., the restriction imposed by the dissipation obstacle and the PDEs to be solved (10), remained unchanged—replacing g by $g\alpha$, which amounts to a redefinition of the plant inputs. In Section V we show that setting $r = m$ and selecting α a function of the plant state x the conditions for Casimir generation are simplified. A discussion on this issue may be found in [34]. See also Remark 7 and point iii) in Section IX.

Remark 4: Necessary and sufficient conditions for the solvability of the PDEs (10), in terms of regularity and involutivity of certain distributions, are given in Proposition 3 of [5].

Remark 5: In [24] the energy shaping action of CbI was viewed from an alternative perspective—geometric instead of Lyapunov–based—that proceeds as follows. First, we notice that the level sets of the Casimir functions, $\zeta - \mathcal{C}(x)$, are invariant sets for the interconnected system. That is, the sets

$$\Omega_\kappa \triangleq \{(x, \zeta) \in \mathbb{R}^n \times \mathbb{R}^m | \zeta = \mathcal{C}(x) + \kappa\}, \ \kappa \in \mathbb{R}$$

are invariant for the overall dynamics.[4] Then, projecting the system on Ω_κ yields the reduced dynamics $\dot{x} = F\nabla H_s$, where $H_s(x) \triangleq H(x) + H_c[\mathcal{C}(x) + \kappa]$ plays the role of shaped energy function. Even though with a proper selection of the initial conditions of the controller we can set $\kappa = 0$, the fact that the shaped energy function depends on this constant is rather unnatural, thus we have presented the result using a Lyapunov approach.

Remark 6: In Proposition 1, and actually throughout most of the paper, we have concentrated on the ability of CbI to shape the energy function, without particular concern of the stability property. Clearly, W will qualify as a Lyapunov function if we can ensure that the desired equilibrium point $(x_\star, \zeta_\star)$ is an isolated minimum of $W(x, \zeta)$. If y is a detectable

output for the overall system [34], *asymptotic* stability of the equilibrium can be enforced adding damping, i.e., setting $v = -K_p y$, $K_p = K_p^\top > 0$, and fixing the initial conditions of the controller states as

$$\zeta(0) = \zeta_\star + \mathcal{C}(x(0)) - \mathcal{C}(x_\star).$$

This initialization is needed to ensure that the trajectory starts (and remains) in the invariant set $\Omega_{\kappa_\star}$, with $\kappa_\star \triangleq \zeta_\star - \mathcal{C}(x_\star)$, that contains the desired equilibrium. See point iv) of Section IX for a discussion on this critical point.

Remark 7: Interestingly, it is possible to show that we cannot generate Casimirs and at the same time add damping through the controller unless we increase the dimension of the dynamic extension, which was taken here to be equal to m. Indeed, replacing $\dot{\zeta} = -\mathcal{R}_c \nabla H_c + u_c$ in (6) and repeating the calculations for the computation of the Casimirs (with $v = 0$) yields the necessary condition $(\nabla \mathcal{C})^\top \mathcal{R} \nabla \mathcal{C} = -\mathcal{R}_c$, which cannot be satisfied with a positive semi–definite matrix $\mathcal{R}_c$. See Section 3.2 of [23] and Example 4.3.3 of [34] for cases where damping propagation from the controller is possible with a dynamic extension of dimension *greater* than m.

C. The Dissipation Obstacle

Proposition 1 shows that, via the selection of H_c and Φ, it is possible to shape the energy function of the interconnected system—provided we can generate Casimir functions. That is, if we can solve the PDEs (10). Unfortunately, the solvability of the latter imposes a serious constraint on the dissipation structure of the system, which was called *dissipation obstacle* in [24].

Proposition 2: If (10) admits a solution then

$$\mathcal{R}\nabla_x \Phi(\mathcal{C}(x) - \zeta) = 0, \tag{13}$$

for all $\Phi : \mathbb{R}^m \to \mathbb{R}$. Consequently, energy cannot be shaped for coordinates that are affected by physical damping.

Proof: Spelling out (10) and combining them we get

$$F^\top \nabla \mathcal{C} = g, \ g^\top \nabla \mathcal{C} = 0 \ \Rightarrow \ (\nabla \mathcal{C})^\top F^\top \nabla \mathcal{C} = 0$$
$$\Rightarrow \ \mathcal{R}\nabla \mathcal{C} = 0.$$

The proof is completed noting that $\nabla_x \Phi = \nabla \mathcal{C} \nabla \Phi$. ∎

It is also possible to express the dissipation obstacle in terms of the energy provided to the plant by the controller. More precisely, we will now show that a PH system with full rank F is stabilizable via CbI *only if* the power extracted from the controller is zero at the equilibrium.

Proposition 3: Let $x_\star$ be the equilibrium of the PH system (1) to be stabilized via CbI, and $u_\star, y_\star$ the corresponding input and output. If (10) admits a solution and F is full rank then $u_\star^\top y_\star = 0$.

Proof: First, note that since $\mathcal{R}\nabla \mathcal{C} = 0$ we have that $F^\top \nabla \mathcal{C} = g$ is equivalent to $F\nabla \mathcal{C} = -g$. Hence, (10) is equivalent to

$$F\nabla \mathcal{C} = -g, \quad g^\top \nabla \mathcal{C} = 0. \tag{14}$$

Consequently, since the inverse of F exists, we have $\nabla \mathcal{C} = -F^{-1}g$, which replaced in $\mathcal{R}\nabla \mathcal{C} = 0$ yields

$$\mathcal{R}F^{-1}g = 0, \tag{15}$$

---

[4]We recall that a set $\Omega_\kappa \subset \mathbb{R}^n \times \mathbb{R}^m$ is invariant if $(x(0), \zeta(0)) \in \Omega_\kappa \Rightarrow (x(t), \zeta(t)) \in \Omega_\kappa$ for all $t \geq 0$. A necessary and sufficient condition for all sets Ω_κ, called the foliation of the manifold Ω_0, to be invariant is precisely $\dot{\zeta} = \dot{\mathcal{C}}$.

that is a *necessary* condition for the existence of Casimirs.

Now, evaluating $\dot{x} = F\nabla H + gu$ at the equilibrium we have the following chain of implications

$$
\begin{aligned}
0 = F_\star \nabla H_\star + g_\star u_\star \;\; &\Leftrightarrow\;\; \nabla H_\star = -F_\star^{-1} g_\star u_\star \\
&\Rightarrow\;\; \mathcal{R}_\star \nabla H_\star = -\mathcal{R}_\star F_\star^{-1} g_\star u_\star \\
&\Rightarrow\;\; \mathcal{R}_\star \nabla H_\star = 0 \\
&\Rightarrow\;\; \nabla^\top H_\star \mathcal{R}_\star \nabla H_\star = 0,
\end{aligned}
$$

where we have invoked (15) to get the third implication. Replacing the latter in the power balance equation (3), and evaluating at the equilibrium, yields the desired result. ∎

Remark 8: It is shown in [33] that the dissipation obstacle is intrinsic, in the sense that it is determined only by the damping interconnection structure and is independent of the actual value of the damping elements.

III. GENERATING NEW CYCLO–PASSIVITY PROPERTIES

To overcome the dissipation obstacle we propose in this section to exploit the non–uniqueness of the PH representation to generate new cyclo–passive outputs. More precisely, we will look for full rank matrices $F_d : \mathbb{R}^n \to \mathbb{R}^{n \times n}$, with

$$
F_d(x) + F_d^\top(x) \le 0, \tag{16}
$$

and storage functions $H_{\mathrm{PS}} : \mathbb{R}^n \to \mathbb{R}$ such that

$$
F(x)\nabla H(x) = F_d(x)\nabla H_{\mathrm{PS}}(x). \tag{17}
$$

It is clear that, if (16) and (17) hold, then the system $\dot{x} = F\nabla H + gu$ with output $g^\top \nabla H_{\mathrm{PS}}$ will be cyclo–passive with storage function H_{PS}. It turns out that $g^\top \nabla H_{\mathrm{PS}}$ is not adequate to overcome the dissipation obstacle and another cyclo–passive output—that, being related with the power shaping procedure of [28], we call y_{PS}—must be generated. Interestingly, we also prove that in the single input case a necessary and sufficient condition for the new cyclo–passive output y_{PS} to be equal to the "natural" output $g^\top \nabla H_{\mathrm{PS}}$ is precisely the absence of the dissipation obstacle.

A. Construction of y_{PS}

The procedure to identify the new cyclo–passive outputs is contained in the following proposition, which requires F_d to be full rank and relies on a direct application of Poincaré's Lemma.[5]

Proposition 4: For all solutions F_d of the PDE

$$
\nabla\left(F_d^{-1} F \nabla H\right) = \left[\nabla\left(F_d^{-1} F \nabla H\right)\right]^\top, \tag{18}
$$

verifying (16) there exists a storage function H_{PS} such that the PH system[6]

$$
\Sigma_{(u, y_{\mathrm{PS}})} \left\{
\begin{aligned}
\dot{x} &= F(x)\nabla H(x) + g(x)u \\
y_{\mathrm{PS}} &= -g^\top(x) F_d^{-\top}(x) F(x) \nabla H(x) - \\
&\quad - g^\top(x) F_d^{-\top}(x) + g(x)u
\end{aligned}
\right. \tag{19}
$$

[5]Poincaré's Lemma: Given $f : \mathbb{R}^n \to \mathbb{R}^n$, $f \in \mathcal{C}^1$. There exists $\psi : \mathbb{R}^n \to \mathbb{R}$ such that $\nabla\psi = f$ if and only if $\nabla f = (\nabla f)^\top$.

[6]We are using here the definition of PH systems with feed–through term introduced in [33].

satisfies the cyclo–passivity inequality

$$
\dot{H}_{\mathrm{PS}} \le u^\top y_{\mathrm{PS}}. \tag{20}
$$

Proof: Poincaré's Lemma states that (18) is necessary and sufficient for the existence of H_{PS} such that

$$
\nabla H_{\mathrm{PS}} = F_d^{-1} F \nabla H, \tag{21}
$$

which is equivalent to (17). We then have the following chain of implications

$$
\begin{aligned}
F_d \nabla H_{\mathrm{PS}} = F \nabla H \;\; &\Rightarrow\;\; \dot{x} = F_d \nabla H_{\mathrm{PS}} + gu \\
&\Leftrightarrow\;\; F_d^{-1} \dot{x} = \nabla H_{\mathrm{PS}} + F_d^{-1} gu \\
&\Rightarrow\;\; \dot{x}^\top F_d^{-1} \dot{x} = \dot{H}_{\mathrm{PS}} + \dot{x}^\top F_d^{-1} gu \\
&\Rightarrow\;\; 0 \ge \dot{H}_{\mathrm{PS}} + \dot{x}^\top F_d^{-1} gu,
\end{aligned}
$$

where the last inequality is obtained using (16) and the fact that $A + A^\top \le 0 \Leftrightarrow A^{-1} + A^{-\top} \le 0$, for any full rank matrix A. The proof is completed replacing $\dot{x}$ and the definition of y_{PS} in (19) in the latter inequality. ∎

Remark 9: Under the assumption that F is full rank we obtain a trivial solution of (18) setting $F_d = F$. In this case, $H_{\mathrm{PS}} = H$ and we obtain the new power–balance equation

$$
\dot{H} = \dot{x}^\top F^{-1} \dot{x} + u^\top y_{\mathrm{PS}}.
$$

Comparing with (3) we see that the new passive output is obtained swapping the damping—as first observed in [15]. In that paper it is also shown that, for electromechanical systems with input voltage sources in series with leaky inductors, y_{PS} results from the application of the classical Thevenin–Norton equivalent of electrical circuits. See also the example in Subsection VIII-B.

Remark 10: The construction proposed in [28] for power–shaping can be used also here to provide solutions of (18), provided F is full rank. Namely, it is easy to show that for all matrices $M : \mathbb{R}^n \to \mathbb{R}^{n \times n}$, with $M(x) = M^\top(x)$ and all $\lambda \in \mathbb{R}$, such that

$$
\tilde{M}(x) \triangleq \frac{1}{2}\left[(\nabla^2 H(x))M(x) + \nabla(M(x)\nabla H(x)) + 2\lambda I_n\right]
$$

is full rank, $F_d^{-1} = \tilde{M} F^{-1}$ solves (18). The resulting storage function being $H_{\mathrm{PS}} = \lambda H + (\nabla H)^\top M \nabla H$.

Remark 11: In [33] it is shown that PH systems with feed–through term take the form

$$
\Sigma_{(u, \tilde{y})} \left\{
\begin{aligned}
\dot{x} &= F(x)\nabla H(x) + g(x)u \\
\tilde{y} &= [g(x) + 2P(x)]^\top \nabla H_{\mathrm{PS}}(x) + \\
&\quad + [T(x) + S(x)]u,
\end{aligned}
\right.
$$

where $S = S^\top$, $T = -T^\top$ and the dissipation structure (defined in $\mathrm{col}(x, u)$) is captured by

$$
\begin{bmatrix} \mathcal{R} & P \\ P^\top & S \end{bmatrix} \ge 0.
$$

Setting, $P = -\frac{1}{2}(I + F_d^\top F_d^{-1})g$, $T = 0$ and $S = -g^\top F_d^{-\top} g$ we see that (19) belongs to this class with $\tilde{y} = y_{\mathrm{PS}}$.

B. When is $y_{\mathrm{PS}} = g^\top \nabla H_{\mathrm{PS}}$? The Role of Dissipation

As indicated above, if (16) and (17) hold, then $g^\top \nabla H_{\mathrm{PS}}$ is a cyclo–passive output and we could apply CbI for the system with the port variables $(u, g^\top \nabla H_{\mathrm{PS}})$. Introducing the natural notation $F_d(x) = \mathcal{J}_d(x) - \mathcal{R}_d(x)$, with $\mathcal{J}_d = -\mathcal{J}_d^\top$ and $\mathcal{R}_d = \mathcal{R}_d^\top \geq 0$, and doing some simple calculations we can prove that in this case a necessary condition for generation of Casimirs is

$$\mathcal{R}_d F_d^{-1} g = 0, \qquad (22)$$

which still imposes a restriction on the damping—compare with (15). We will show in the next section that applying CbI to y_{PS}, instead of $g^\top \nabla H_{\mathrm{PS}}$, this restriction is removed. Interestingly, the proposition below proves that the construction of Proposition 4 will generate new passive outputs if and only if (22) does not hold.[7]

We require the following basic lemma.

Lemma 1: $g^\top F_d^{-\top} g = -g^\top F_d^{-\top} \mathcal{R}_d F_d^{-1} g.$

Proof: We compute

$$
\begin{aligned}
g^\top F_d^{-\top} g &= \frac{1}{2} g^\top (F_d^{-\top} + F_d^{-1}) g \\
&= \frac{1}{2} g^\top F_d^{-\top} (F_d + F_d^\top) F_d^{-1} g.
\end{aligned}
$$

The proof is completed with $F_d + F_d^\top = -2\mathcal{R}_d$. ∎

Proposition 5: In the single input single output case the new cyclo–passive output y_{PS} is equal to $g^\top \nabla H_{\mathrm{PS}}$ if and only if the dissipation obstacle for the PH system with port variables $(u, g^\top \nabla H_{\mathrm{PS}})$ is absent, that is

$$
\begin{aligned}
\mathcal{R}_d F_d^{-1} g &= 0 \\
&\Leftrightarrow \\
g^\top \nabla H_{\mathrm{PS}} &= -g^\top F_d^{-\top} (F_d \nabla H_{\mathrm{PS}} + gu) \quad (= y_{\mathrm{PS}}).
\end{aligned}
$$

Proof: From the definition of y_{PS} in (19) and (17) we have

$$
\begin{aligned}
y_{\mathrm{PS}} &= -g^\top F_d^{-\top} (F_d \nabla H_{\mathrm{PS}} + gu) \pm g^\top \nabla H_{\mathrm{PS}} \\
&= -g^\top F_d^{-\top} (F_d + F_d^\top) \nabla H_{\mathrm{PS}} - g^\top F_d^{-\top} gu + \\
&\quad + g^\top \nabla H_{\mathrm{PS}} \\
&= g^\top F_d^{-\top} \mathcal{R}_d (2 \nabla H_{\mathrm{PS}} + F_d^{-1} gu) + g^\top \nabla H_{\mathrm{PS}},
\end{aligned}
$$

where we have added and subtracted $g^\top \nabla H_{\mathrm{PS}}$ in the first line and invoked Lemma 1 to obtain the third identity. The proof is completed noting that the sum of the first and the second right hand term in the last equation is zero if and only if $\mathcal{R}_d F_d^{-1} g = 0$. ∎

Remark 12: Setting $F_d = F$ we obtain as a simple corollary of Proposition 5 the equivalence

$$\mathcal{R} F^{-1} g = 0 \Leftrightarrow y = y_{\mathrm{PS}}. \qquad (23)$$

The sufficiency part of this equivalence had been established before in [16].

---

[7]In this subsection we assume that the system is single input, e.g., $m = 1$, in this case, g is a column vector. For the multi–input case the condition is only sufficient.

IV. CONTROL BY INTERCONNECTION WITH $\Sigma_{(u, y_{\mathrm{PS}})}$

In this section we apply the CbI methodology to the new PH system $\Sigma_{(u, y_{\mathrm{PS}})}$ and show that, in this way, we can shape even the coordinates where dissipation is present. More precisely, we will remove the second condition for existence of Casimirs in (10), obviating the dissipation obstacle (13). To differentiate this controller from the one obtained using $\Sigma_{(u, y)}$ we refer to it as CbI_{PS}. Moreover, we distinguish two variations, when $F_d = F$, that we call Basic CbI_{PS}, and when $F_d \neq F$ that we refer as CbI_{PS}.

A. CbI_{PS} Overcomes the Dissipation Obstacle

Proposition 6: Assume the PDE (18) admits a solution F_d verifying (16) and such that

$$F_d \nabla \mathcal{C} = -g, \qquad (24)$$

for some vector function $\mathcal{C} : \mathbb{R}^n \to \mathbb{R}^m$. Consider the PH system (19) coupled with the PH controller Σ_c (6) through the power–preserving interconnection subsystem

$$
\Sigma_I^{\mathrm{PS}} : \left\{ \begin{bmatrix} u \\ u_c \end{bmatrix} = \begin{bmatrix} 0 & -I_m \\ I_m & 0 \end{bmatrix} \begin{bmatrix} y_{\mathrm{PS}} \\ y_c \end{bmatrix} + \begin{bmatrix} v \\ 0 \end{bmatrix} \right.. \qquad (25)
$$

Then, for all functions $\Phi : \mathbb{R}^m \to \mathbb{R}$, the following cyclo–passivity inequality is satisfied

$$\dot{W}_{\mathrm{PS}} \leq v^\top y_{\mathrm{PS}}, \qquad (26)$$

where the storage function $W_{\mathrm{PS}} : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ is defined as

$$W_{\mathrm{PS}}(x, \zeta) \triangleq H_{\mathrm{PS}}(x) + H_c(\zeta) + \Phi(\mathcal{C}(x) - \zeta), \qquad (27)$$

with $H_{\mathrm{PS}} = \int (F_d^{-1} F \nabla H) dx$.

Proof: The proof directly mimics the proof of Proposition 1. The dynamics of the interconnected system are described by

$$
\begin{bmatrix} \dot{x} \\ \dot{\zeta} \end{bmatrix} = \begin{bmatrix} F_d & -g \\ -g^\top F_d^{-\top} F_d & g^\top F_d^{-\top} g \end{bmatrix} \begin{bmatrix} \nabla H_{\mathrm{PS}} \\ \nabla H_c \end{bmatrix} + \\ + \begin{bmatrix} g \\ -g^\top F_d^{-\top} g \end{bmatrix} v.
$$

Computing the time derivatives

$$
\begin{aligned}
\dot{\mathcal{C}} - \dot{\zeta} &= \begin{bmatrix} (\nabla \mathcal{C})^\top & | & -I_m \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\zeta} \end{bmatrix} \\
&= \begin{bmatrix} -g^\top F_d^{-\top} & | & -I_m \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\zeta} \end{bmatrix} = 0.
\end{aligned}
$$

where the second equation is obtained from (24), and the last equation holds for all H_{PS}, H_c. Hence, $\dot{\Phi} = 0$. This, together with (7), (20) and (27) completes the proof. ∎

Remark 13: The key difference between Propositions 1 and 6 is that the second condition for generation of Casimirs in the former, namely $g^\top \nabla \mathcal{C} = 0$, is conspicuously absent in the latter. As pointed out in Subsection II-C if both conditions in (10) are satisfied then the dissipation obstacle condition for CbI appears—see also (15). This restriction is not imposed in CbI_{PS}.

Remark 14: In [20] the cyclo–passive output y_{PS} was obtained, in the context of stability analysis of PH systems, with

the following alternative construction. Suppose we can find $\mathcal{C}$ satisfying

$$F\nabla\mathcal{C} + g = 0. \tag{28}$$

Construct now the interconnection and dissipation matrices of an *augmented system* as

$$\mathcal{J}_a \triangleq \left[\begin{array}{cc} \mathcal{J} & \mathcal{J}\nabla\mathcal{C} \\ (\nabla\mathcal{C})^\top\mathcal{J} & (\nabla\mathcal{C})^\top\mathcal{J}\nabla\mathcal{C} \end{array} \right] = -\mathcal{J}_a^\top,$$

$$\mathcal{R}_a \triangleq \left[\begin{array}{cc} \mathcal{R} & \mathcal{R}\nabla\mathcal{C} \\ (\nabla\mathcal{C})^\top\mathcal{R} & (\nabla\mathcal{C})^\top\mathcal{R}\nabla\mathcal{C} \end{array} \right] = \mathcal{R}_a^\top \geq 0.$$

By construction

$$\left[\begin{array}{cc} (\nabla\mathcal{C})^\top & | & -I_m \end{array} \right] \mathcal{J}_a = \left[\begin{array}{cc} (\nabla\mathcal{C})^\top & | & -I_m \end{array} \right] \mathcal{R}_a = 0,$$

implying that $\mathcal{C} - \zeta$ are Casimirs for the PH dynamics

$$\left[\begin{array}{c} \dot{x} \\ \dot{\zeta} \end{array} \right] = (\mathcal{J}_a - \mathcal{R}_a) \left[\begin{array}{c} \nabla H(x) \\ \nabla H_c(\zeta) \end{array} \right].$$

Furthermore, because of (28),

$$\mathcal{J}_a - \mathcal{R}_a = \left[\begin{array}{cc} \mathcal{J} - \mathcal{R} & -g \\ (g - 2\mathcal{R}\nabla\mathcal{C})^\top & (\nabla\mathcal{C})^\top(\mathcal{J} - \mathcal{R})\nabla\mathcal{C} \end{array} \right].$$

Thus, the augmented systems is the unitary feedback interconnection of the nonlinear integrators (6) with the PH plant with a different output, that turns out to be y_{PS} for $F_d = F$! It is interesting to note that these derivations do not presume the invertibility of F.

Remark 15: From the definition of y_{PS} in (19) and (24) we see that, if the Casimirs exist, $y_{\mathsf{PS}} = \dot{\mathcal{C}}$, which in its turn is equal to $\dot{\zeta}$. Hence, if we introduce the partial change of coordinates $z = \mathcal{C}(x) - \zeta$, we get $\dot{z} = 0$. This is another way of viewing that the controller is rendering all the sets Ω_κ invariant. See Remark 6.

V. CONTROL BY STATE–MODULATED INTERCONNECTION

In this section we will replace the simple negative feedback interconnection Σ_I by a state–modulated interconnection [34], as suggested in Remark 3. In this way we will further relax the condition for existence of Casimirs: (10) for the CbI of Section II, and (24) for the CbI_{PS} of Section IV. We will call the new controllers CbI^{SM} for the former and, for the controllers using y_{PS}, Basic $CbI_{\mathsf{PS}}^{\mathsf{SM}}$ if $F_d = F$ and $CbI_{\mathsf{PS}}^{\mathsf{SM}}$ if $F_d \neq F$.

The following elementary, though somehow overlooked, result will be used in the sequel.

Lemma 2: Let $g \in \mathbb{R}^{n \times m}$, $m < n$ with rank $g = m$. Define $g^\perp \in \mathbb{R}^{(n-m) \times n}$ as a full rank left annihilator of g, that is, $g^\perp g = 0$ and rank $\{g^\perp\} = n - m$. For any $b \in \mathbb{R}^n$, $\hat{u} \in \mathbb{R}^m$

$$b + g\hat{u} = 0 \quad \Leftrightarrow \quad \left\{ \begin{array}{rcl} g^\perp b &=& 0 \\ \hat{u} &=& -(g^\top g)^{-1}g^\top b \end{array} \right. \tag{29}$$

Proof: The matrix $\left[\begin{array}{c} g^\perp \\ g^\top \end{array} \right] \in \mathbb{R}^{n \times n}$ is full rank. Hence,

$$b + g\hat{u} = 0 \quad \Leftrightarrow \quad \left[\begin{array}{c} g^\perp \\ g^\top \end{array} \right] (b + g\hat{u}) = 0.$$

The proof is completed using the annihilating property of $g^\perp$ and noting that the square matrix $g^\top g$ is full rank. ∎

A. Energy Shaping via CbI^{SM}

Proposition 7: Assume the PDE

$$\left[\begin{array}{c} g^\perp F^\top \\ g^\top \end{array} \right] \nabla\mathcal{C} = 0, \tag{30}$$

admits a solution for some vector function $\mathcal{C} : \mathbb{R}^n \to \mathbb{R}^m$. Consider the PH system $\Sigma_{(u,y)}$ (1) coupled with the PH controller Σ_c (6) through the *state–modulated* power–preserving interconnection

$$\Sigma_I^{\mathsf{SM}} : \left\{ \left[\begin{array}{c} u \\ u_c \end{array} \right] = \left[\begin{array}{cc} 0 & -\alpha(x) \\ \alpha^\top(x) & 0 \end{array} \right] \left[\begin{array}{c} y \\ y_c \end{array} \right] + \left[\begin{array}{c} v \\ 0 \end{array} \right], \right. \tag{31}$$

where $\alpha : \mathbb{R}^n \to \mathbb{R}^{m \times m}$ is defined as

$$\alpha = -(g^\top g)^{-1}g^\top F^\top \nabla\mathcal{C}. \tag{32}$$

Then, for all functions $\Phi : \mathbb{R}^m \to \mathbb{R}$, the cyclo–passivity inequality (12) with storage function (11) is satisfied.

Proof: The proof goes along the same lines as the proof of Proposition 1, therefore is only sketched here. The dynamics of the interconnected system is given by

$$\left[\begin{array}{c} \dot{x} \\ \dot{\zeta} \end{array} \right] = \left[\begin{array}{cc} F & -g\alpha \\ \alpha^\top g^\top & 0 \end{array} \right] \left[\begin{array}{c} \nabla H \\ \nabla H_c \end{array} \right] + \left[\begin{array}{c} g \\ 0 \end{array} \right] v.$$

Computing $\dot{\mathcal{C}} - \dot{\zeta}$, and noting that, in view of Lemma 2, $g^\perp F^\top \nabla\mathcal{C} = 0$ and (32) are equivalent to $F^\top \nabla\mathcal{C} = -g\alpha$, completes the proof. ∎

Remark 16: It is clear that the set of solutions of (30) is strictly larger than the one of (10). Indeed, (30) is necessary, but not sufficient, for (10). The inclusion of state modulation in the interconnection has allowed, through the addition of the matrix α, to significantly extend the class of systems for which the CbI method is applicable. However, it is easy to show that the controller above still suffers from the dissipation obstacle, namely: (30) $\Rightarrow \mathcal{R}\nabla\mathcal{C} = 0$.

B. Energy Shaping via $CbI_{\mathsf{PS}}^{\mathsf{SM}}$

A similar result is obtained for CbI_{PS}, whose proof is omitted for brevity.

Proposition 8: Assume the PDE (18) admits a solution F_d verifying (16) and such that

$$g^\perp F_d \nabla\mathcal{C} = 0, \tag{33}$$

for some vector function $\mathcal{C} : \mathbb{R}^n \to \mathbb{R}^m$. Consider the PH system (19) coupled with the PH controller Σ_c (6) through the *state–modulated* power–preserving interconnection subsystem

$$\Sigma_I^{\mathsf{PS-SM}} : \left\{ \left[\begin{array}{c} u \\ u_c \end{array} \right] = \left[\begin{array}{cc} 0 & -\alpha(x) \\ \alpha^\top(x) & 0 \end{array} \right] \left[\begin{array}{c} y_{\mathsf{PS}} \\ y_c \end{array} \right] + \left[\begin{array}{c} v \\ 0 \end{array} \right], \right. \tag{34}$$

where $\alpha : \mathbb{R}^n \to \mathbb{R}^{m \times m}$ is defined as

$$\alpha = -(g^\top g)^{-1}g^\top F_d \nabla\mathcal{C}. \tag{35}$$

Then, for all functions $\Phi : \mathbb{R}^m \to \mathbb{R}$, the cyclo–passivity inequality (26) with storage function (27) is satisfied.

VI. STANDARD PASSIVITY–BASED CONTROL REVISITED

In [24] we introduced the following:

Definition 1: Consider the PH system (1) verifying the power–balance equation (3), that we repeat here for ease of reference

$$\dot{H} = u^\top y - d,$$

with $d \triangleq (\nabla H)^\top \mathcal{R} \nabla H \geq 0$ the open–loop dissipation. A control action $u = \hat{u}(x) + v$ solves the *Standard PBC problem* if the closed–loop system satisfies the desired power–balance equation

$$\dot{H}_d = v^\top z - d_d, \qquad (36)$$

where $H_d : \mathbb{R}^n \to \mathbb{R}_+$ is the desired energy function, $d_d : \mathbb{R}^n \to \mathbb{R}_+$ is the desired damping, and $z \in \mathbb{R}^m$ is a new passive output.

The problem above has too many "degrees of freedom", i.e., $H_d, d_d, z, \hat{u}$.[8] In spite of this, in the present section we derive from a *unified perspective* four solutions to this problem. Namely, we will show that selecting various desired dissipation functions, d_d, generates different versions of Standard PBC, which were previously obtained independently invoking other considerations. The definition below is instrumental to streamline our results.

Definition 2: Define the *added* energy function

$$H_a(x) \triangleq H_d(x) - H(x). \qquad (37)$$

A state feedback that solves the Standard PBC problem satisfies the Energy–Balancing (EB) property—for short, is EB—if the added energy H_a equals the energy supplied to the system by the environment, that is, if

$$\dot{H}_a = -\hat{u}^\top y. \qquad (38)$$

Consequently, the total energy function H_d is the difference between the stored and the supplied energies.

A. Preliminary Results and Proposed Approach

Before presenting the main results of the section we find convenient to recall the fundamental Hill–Moylan's Lemma [11] whose proof, in the present formulation, may be found in [32]. We also present a corollary to Hill–Moylan's Lemma, that is instrumental for the solution of the Standard PBC problem, as well as the proposed approach.

Lemma 3: The system

$$\dot{x} = f(x) + g(x)u, \ y = h(x)$$

is cyclo–passive with storage function $V : \mathbb{R}^n \to \mathbb{R}$, $V \in \mathcal{C}^1$, i.e. $\dot{V} \leq u^\top y$, *iff* there exists a *damping function* $d : \mathbb{R}^n \to \mathbb{R}_+$ such that[9]

$$\nabla V^\top f = -d \qquad (39)$$
$$g^\top \nabla V = h. \qquad (40)$$

---

[8]To ensure stability of an equilibrium $x_\star$ we impose $x_\star = \arg \min H_d$, and for asymptotic stability z should be a detectable output—see Remark 6.

[9]Condition (40) justifies the choice of output made in Remark 2. It is important o underscore that d is a function of the state.

Corollary 1: Consider the PH system $\Sigma_{(u,y)}$ (1) in closed–loop with $u = \hat{u}(x) + v$. Then (36) holds iff

$$\nabla H_d^\top (F \nabla H + g \hat{u}) = -d_d \qquad (41)$$
$$z = g^\top \nabla H_d, \qquad (42)$$

for some function $d_d : \mathbb{R}^n \to \mathbb{R}_+$.

In view of Corollary 1, that fixes the new passive output z via (42), our problem is now to find $(H_d, d_d, \hat{u})$ that will solve (41) for a given triple (F, g, H). We propose to select the desired damping d_d to be able to define a control signal $\hat{u}$—function of H_d—so that (41) becomes a *linear PDE* in the unknown *assignable* energy functions H_d. For solvability purposes, the qualifier "linear" in the PDE is essential in the procedure.

Remark 17: For linear time–invariant systems, $\dot{x} = Ax + Bu$, with

$$\hat{u} = Kx, \quad H_d = \frac{1}{2} x^\top P_d x, \quad d_d = \frac{1}{2} x^\top R_d x$$

(41) becomes the Lyapunov equation

$$P_d(A + BK) + (A + BK)^\top P_d = -R_d.$$

Remark 18: A version of Hill–Moylan's Lemma for systems with direct throughput may be found in [11], [32]. For simplicity, we have decided to consider systems without throughput. This is done without loss of generality because, for our purposes, the key equation to be verified is (39) that remains unchanged.

B. Energy–Balancing PBC

Proposition 9: Fix $d_d = d = -\nabla H^\top F \nabla H$, and denote $\hat{u} = \hat{u}_{\text{EB}}$.

(i) The control law $\hat{u}_{\text{EB}} = -(g^\top g)^{-1} g^\top F^\top \nabla H_a$, with H_a solution of the PDEs

$$\begin{bmatrix} g^\perp F^\top \\ g^\top \end{bmatrix} \nabla H_a = 0, \qquad (43)$$

solves the Standard PBC problem.

(ii) The controller is EB, that is, (38) holds.

(iii) EB–PBC suffers from the *dissipation obstacle*. More precisely,

$$(43) \Rightarrow \mathcal{R} \nabla H_a = 0. \qquad (44)$$

Proof: We will verify that (41) holds. Thus,

$$(41) \text{ with } d_d = d \Leftrightarrow \nabla H_d^\top (F \nabla H + g \hat{u}_{\text{EB}}) = \nabla H^\top F \nabla H$$
$$\Leftrightarrow \nabla H_d^\top g \hat{u}_{\text{EB}} = -\nabla H_a^\top F \nabla H$$
$$\Leftrightarrow \nabla H^\top g \hat{u}_{\text{EB}} = -\nabla H_a^\top F \nabla H$$
$$\Leftrightarrow \nabla H^\top (F^\top \nabla H_a + g \hat{u}_{\text{EB}}) = 0, \qquad (45)$$

where we used $g^\top \nabla H_a = 0$ to obtain the third equivalence. Applying Lemma 2 to the term in parenthesis we get the proposed solution

$$g^\perp F^\top \nabla H_a = 0, \quad \hat{u}_{\text{EB}} = -(g^\top g)^{-1} g^\top F^\top \nabla H_a.$$

To establish the EB property we have

$$g^\top \nabla H_a = 0 \Rightarrow z = y$$
$$\Rightarrow \dot{H}_d = y^\top v - d$$
$$\Rightarrow \dot{H}_a = -y^\top \hat{u}_{\text{EB}},$$

where we have used $z = g^\top \nabla H_d$ in the first implication, $d_d = d$ in the second and $\dot{H} = y^\top(\hat{u}_{\text{EB}} + v) - d$ for the last one. Finally, from $F^\top \nabla H_a = -g\hat{u}_{\text{EB}}$, premultiplying by $\nabla H_a^\top$, and using $g^\top \nabla H_a = 0$ yields $\nabla H_a^\top F^\top \nabla H_a = 0$, which is equivalent to $\mathcal{R}\nabla H_a = 0$. ∎

Remark 19: EB–PBC are widely popular for potential energy shaping of mechanical systems. In this case

$$x = \begin{bmatrix} q \\ p \end{bmatrix}, \quad H(q,p) = \frac{1}{2}p^\top M^{-1}(q)p + V(q),$$

$$F = \begin{bmatrix} 0 & I \\ -I & -R \end{bmatrix}, \quad g(q) = \begin{bmatrix} 0 \\ G(q) \end{bmatrix},$$

and the added energy is $H_a(q) = V_d(q) - V(q)$, where (q,p) are the generalized coordinates and momenta, $M = M^\top > 0$ is the inertia matrix, $R = R^\top \geq 0$ is the dissipation due to friction, G is the input matrix and V, V_d are the open–loop and desired potential energies, respectively. Some simple calculations show that (43) becomes $G^\perp(\nabla V_d - \nabla V) = 0$, which is known as the potential energy matching equation [3], [25].

Remark 20: The restriction imposed by the dissipation captured by (44) is of the same nature as the one imposed to CbI, namely, (13). In both cases, we are unable to shape the coordinates where dissipation is directly present. In Subsection III-B we proved that the construction of y_{PS} used for CbI_{PS} yielded the same output, i.e., $y_{\text{PS}} = y$, iff the dissipation obstacle is absent—that is, when there is no need for the new output! Interestingly, we will show in the next subsection that Standard PBCs that do not suffer from this limitation will be EB, precisely if the dissipation obstacle is absent. In other words, for both CbI and Standard PBC, our ability to ensure that the difference between the energies is a non–increasing function is determined by the nature of the dissipation.

Remark 21: In [24] EB–PBC was derived looking for functions H_a and $\hat{u}$ that satisfy (38). This is, of course, equivalent to solving the PDE $(\nabla H_a)^\top(F\nabla H + gu_{\text{EB}}) = u_{\text{EB}}^\top g^\top \nabla H$, which is the first line in (45).

C. Interconnection and Damping Assignment PBC

We derive in the propositions below the two versions of IDA–PBC reported in [27]: when the interconnection and damping matrices are left unchanged, called Basic IDA–PBC, and when they are modified, that we simply call IDA–PBC. As shown in [27], neither one of the schemes is limited by the dissipation obstacle. The proofs of the propositions, being similar to the proof of Proposition 9, are omitted for the sake of brevity.

Proposition 10: Fix $d_d = -\nabla H_d^\top F\nabla H_d$, and denote $\hat{u} = \hat{u}_{\text{BIDA}}$.
(i) The control law $\hat{u}_{\text{BIDA}} = (g^\top g)^{-1}g^\top F\nabla H_a$, with H_a solution of the PDE $g^\perp F\nabla H_a = 0$, solves the Standard PBC problem.
(ii) If $v = 0$ and there is no dissipation obstacle, i.e., if $\mathcal{R}\nabla H_a = 0$ then Basic IDA–PBC is EB.

Proposition 11: Fix $d_d = -\nabla H_d^\top F_d \nabla H_d$ with $F_d + F_d^\top \leq 0$, and denote $\hat{u} = \hat{u}_{\text{IDA}}$.
(i) The control law

$$\hat{u}_{\text{IDA}} = (g^\top g)^{-1}g^\top[F_d\nabla H_a + (F_d - F)\nabla H],$$

with H_a solution of the PDE

$$g^\perp F_d\nabla H_a = g^\perp(F - F_d)\nabla H, \qquad (46)$$

solves the Standard PBC problem.
(ii) If $v = 0$, the damping is left unchanged and there is no dissipation obstacle, i.e.,

$$\mathcal{R} = -\frac{1}{2}(F_d + F_d^\top), \quad \mathcal{R}\nabla H_a = 0,$$

then IDA–PBC is EB.

Remark 22: Applying Lemma 2 to the equations in point (i) of Proposition 10 we conclude that $F\nabla H_a = g\hat{u}_{\text{BIDA}}$, hence the closed–loop system for Basic IDA–PBC is $\dot{x} = F\nabla H_d + gv$, that is, only the energy is shaped. On the other hand, proceeding analogously for IDA–PBC we have that the closed–loop is now $\dot{x} = F_d\nabla H_d + gv$, where F_d contains the desired interconnection and damping matrices—motivating the name IDA.

D. Power–Shaping PBC

Let us briefly recall the methodology of Power Shaping (PS) PBC that was introduced in [28] as an alternative to energy shaping PBC for stabilization of nonlinear RLC circuits, and was later extended for general nonlinear systems of the form $\dot{x} = f(x) + g(x)u$ in [9]. The name, Power Shaping, was motivated by the fact that, in the case of RLC circuits, the storage functions have *units of power*, as opposed to energy as is normally the case in PBC of PH systems.

The starting point for PS–PBC of RLC circuits is to describe the system using, so–called, Brayton–Moser models [4] where the state coordinates are the co–energy variables (voltages in capacitors and currents in inductors) as opposed to energy variables (charges in capacitors and fluxes in inductors), which are used in PH models. With this choice of state variables it is possible to show that, for a large class of nonlinear RLC circuits, the dynamics are described by[10]

$$Q(x)\dot{x} = \nabla P(x) + Bu, \qquad (47)$$

where $u \in \mathbb{R}^m$ consists of voltage and current sources, $Q : \mathbb{R}^n \to \mathbb{R}^{n \times n}$ is a full rank block diagonal matrix containing the generalized inductance and the generalized capacitance matrices, and $P : \mathbb{R}^n \to \mathbb{R}$—which has units of power, and is called the mixed potential function—captures the interconnection structure and the dissipation. This should be contrasted with PH models, where F contains the interconnection and damping matrices and H is the energy function.[11]

---

[10] To avoid cluttering we use the same symbol, x, to denote the new state variables.

[11] Relationships between the two descriptions have been studied in [13]. See also [33] for a general procedure to transform from one model to the other via the Legendre transform. See also the example in Subsection VIII-B.

Stabilization via PS–PBC proceeds in two steps, first, the selection of a pair $(\tilde{Q}, \tilde{P})$ such that,

$$Q^{-1}\nabla P = \tilde{Q}^{-1}\nabla\tilde{P}$$

$$\left(\Leftrightarrow \nabla(\tilde{Q}Q^{-1}\nabla P) = [\nabla(\tilde{Q}Q^{-1}\nabla P)]^\top\right) \quad (48)$$

with $\tilde{Q} + \tilde{Q}^\top \leq 0$. In this way, we can prove that the system can be written in the form

$$\tilde{Q}\dot{x} = \nabla\tilde{P} + \tilde{Q}Q^{-1}Bu,$$

and clearly satisfies the cyclo–passivity inequality[12]

$$\dot{\tilde{P}} \leq u^\top\tilde{y}, \quad \tilde{y} \overset{\triangle}{=} B^\top Q^{-\top}\tilde{Q}^\top\dot{x}.$$

This first step is, obviously, identical to the procedure for generation of y_{PS} of Proposition 4. More precisely, identifying $F = Q^{-1}$, $F_d = \tilde{Q}^{-1}$ and $H_{\text{PS}} = \tilde{P}$ equation (48) coincides with (18).

In the second step we shape the power function $\tilde{P}$ by adding a function $P_a : \mathbb{R}^n \to \mathbb{R}$, solution of the PDE

$$B^\perp Q\tilde{Q}^{-1}\nabla P_a = 0, \quad (49)$$

which, together with a suitably defined control, yields the closed–loop dynamics $\tilde{Q}\dot{x} = \nabla(\tilde{P} + P_a)$. Identifying $g = Q^{-1}B$ and F_d as above, the PDE (49) reduces to (33), proving the equivalence of PS–PBC and CbI_{PS}.

PS–PBC can also be derived, like the previous Standard PBCs, fixing a desired dissipation. Again, in the interest of brevity, we omit the proof of the proposition.

Proposition 12: Consider the solutions F_d, with $F_d + F_d^\top \leq 0$, of (18). Fix $d_d = -(F\nabla H + g\hat{u}_{\text{PS}})^\top F_d^{-1}(F\nabla H + g\hat{u}_{\text{PS}})$, and denote $\hat{u} = \hat{u}_{\text{PS}}$. The control law $\hat{u}_{\text{PS}} = (g^\top g)^{-1}g^\top F_d\nabla\tilde{H}_a$, with $\tilde{H}_a$ solution of the PDE $g^\perp F_d\nabla\tilde{H}_a = 0$ and $\tilde{H}_d(x) \overset{\triangle}{=} H_a(x) + H_{\text{PS}}(x)$, solves the Standard PBC problem.

Remark 23: It is also possible to relate PS–PBC and IDA–PBC, viewing the former as a two step procedure to solve the PDE of IDA–PBC, (46), which can be written as $g^\perp F_d\nabla H_d = g^\perp F\nabla H$. While in IDA–PBC we *fix* F_d, in PS–PBC we obtain it from the solution of (18). This ensures $F\nabla H = F_d\nabla H_{\text{PS}}$, which replaced in the equation above yields $g^\perp F_d\nabla\tilde{H}_a = 0$. It is important to note that (46) may have solutions even though $F_d^{-1}F\nabla H$ is not a gradient of some function—as required by (18).

VII. CBI AND STANDARD PBC: RELATIONSHIPS AND COMPARISONS

In this section we relate and compare CbI and Standard PBC using three different criteria.

i) Comparing the "size" of the set of PH plants for which they are applicable—this is determined by the "size" of the solution set of the PDEs that need to be solved for each of the methods.

ii) Proving that the (static feedback) Standard PBC laws are the restriction of the (dynamic feedback) CbI to the invariant sets defined by the Casimir functions.

iii) Showing that if CbI can stabilize a given plant then this is also possible with the corresponding Standard PBC—showing that, from the stabilization viewpoint, there is no advantage in considering dynamic feedback.

A. Domain of Applicability

We find convenient to recall the PDEs that need to be solved for each one of the PBC methods.[13]

Control by Interconnection

- (CbI)

$$\begin{bmatrix} F \\ g^\top \end{bmatrix}\nabla\mathcal{C} = \begin{bmatrix} -g \\ 0 \end{bmatrix}.$$

- (CbI^{SM})

$$\begin{bmatrix} g^\perp F \\ g^\top \end{bmatrix}\nabla\mathcal{C} = 0.$$

- (Basic CbI_{PS})

$$F\nabla\mathcal{C} = -g.$$

- (Basic $CbI_{\text{PS}}^{\text{SM}}$)

$$g^\perp F\nabla\mathcal{C} = 0.$$

- (CbI_{PS})

$$F_d\nabla\mathcal{C} = -g,$$

plus (18).
- $(CbI_{\text{PS}}^{\text{SM}})$

$$g^\perp F_d\nabla\mathcal{C} = 0,$$

plus (18).

Standard PBC

- (EB)

$$\begin{bmatrix} g^\perp F \\ g^\top \end{bmatrix}\nabla H_a = 0.$$

- (Basic IDA)

$$g^\perp F\nabla H_a = 0.$$

- (PS)

$$g^\perp F_d\nabla H_a = 0,$$

plus (18).
- (IDA)

$$g^\perp F_d\nabla H_a = g^\perp(F - F_d)\nabla H.$$

The relationship between all these schemes is summarized in the implications diagram of Fig. 2. The notation $A \to B$ means that the set of solutions of the PDEs of B is strictly larger than the one of A, consequently the set of plants to which B is applicable is also strictly larger. Also, we say $A \leftrightarrow B$ if the PDEs are the same. We observe that, in this sense, the more general method is IDA–PBC that has no "CbI version".

---

[12] In the Brayton–Moser model for RLC circuits the matrix Q is sign indefinite, hence this step is needed to establish the cyclo–passivity.

[13] We recall that we defined $\mathcal{C} : \mathbb{R}^n \to \mathbb{R}^m$, while $H_a : \mathbb{R}^n \to \mathbb{R}$. However, in the light of Remark 3, we can always take the order of the dynamic extension to be one, and $\mathcal{C}$ will be a scalar function.
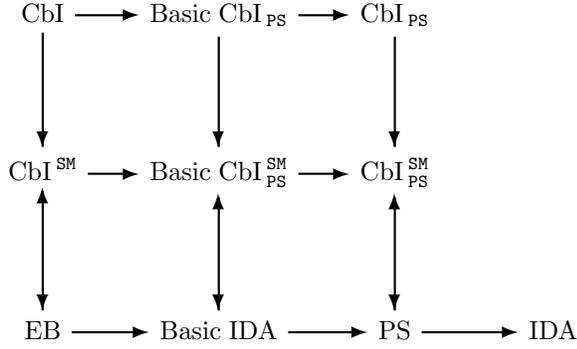
Fig. 2. Relationship between the different control schemes from the point of view of domain of applicability.

B. Standard PBC as a Restriction of CbI

The following proposition shows that, restricting the dynamics of CbI to the set Ω_0, yields an EB–PBC.

Proposition 13: Assume the PDEs (10) admit a solution. Then, for all functions $H_c : \mathbb{R}^m \to \mathbb{R}$, the PH system $\Sigma_{(u,y)}$ (1) in closed–loop with the static state–feedback control $u = \hat{u}_{\text{EB}}(x) + v$, where $\hat{u}_{\text{EB}}(x) = -\nabla_{\mathcal{C}} H_c(\mathcal{C}(x))$, satisfies the cyclo–passivity inequality

$$\dot{H}_d \leq v^\top y, \qquad (50)$$

where $H_d = H + H_a$ with[14]

$$H_a(x) \overset{\triangle}{=} H_c(\mathcal{C}(x)). \qquad (51)$$

Furthermore, the controller is EB.

Proof: Computing from (51) the time derivative

$$
\begin{aligned}
\dot{H}_a &= (\nabla_{\mathcal{C}} H_c(\mathcal{C}))^\top (\nabla \mathcal{C})^\top (F \nabla H + g u) \\
&= (\nabla_{\mathcal{C}} H_c(\mathcal{C}))^\top g^\top \nabla H \\
&= -\hat{u}_{\text{EB}}^\top y,
\end{aligned}
$$

where the second identity is obtained using (10) and the last one replacing $\hat{u}_{\text{EB}}$ and the definition of y. This establishes the EB claim. The cyclo–passivity inequality (50) follows replacing $u = \hat{u}_{\text{EB}}(x) + v$ in (4), using the definition of H_d and the last identity above. ∎

Similarly to CbI, CbI_{PS} also admits a static state feedback realization. Now, the resulting control law and storage function are solutions of the matching equation of IDA–PBC.

Proposition 14: Assume the PDEs of CbI_{PS}, (18) and (24), are satisfied. Then, for all $H_c : \mathbb{R}^m \to \mathbb{R}$, the state–feedback controller $\hat{u}_{\text{IDA}}(x) = -\nabla_{\mathcal{C}} H_c(\mathcal{C}(x))$, ensures that the IDA–PBC matching condition

$$F \nabla H + g \hat{u}_{\text{IDA}} = F_d \nabla H_d \qquad (52)$$

is satisfied with $H_d = H_{\text{PS}} + H_a$ and H_a given by (51).

Proof: For ease of reference, we repeat here the PDEs of CbI_{PS}:

$$
\begin{aligned}
F \nabla H &= F_d \nabla H_{\text{PS}} \\
F_d \nabla \mathcal{C} &= -g.
\end{aligned}
$$

---

[14] H_d is, up to an additive constant, the restriction of W (11) to the set $\zeta = \mathcal{C}(x)$. Clearly, the "free" functions Φ and H_c play the same role in the energy–shaping—as will be further clarified in Subsection VII-C.

Replacing in the matching equation (52) yields

$$F_d[\nabla H_{\text{PS}} - (\nabla \mathcal{C}) \hat{u}_{\text{IDA}}] = F_d \nabla H_d \;\Leftrightarrow\; \nabla H_a = -(\nabla \mathcal{C}) \hat{u}_{\text{IDA}},$$

which is satisfied with the expressions of H_a and $\hat{u}_{\text{IDA}}$ given in the proposition. ∎

C. Stabilization via CbI ⇒ Stabilization via Standard PBC

Throughout the paper we have concentrated our attention on the ability of the various PBCs to modify the energy function, without particular concern to stabilization. As indicated above, stability will be ensured if a strict minimum is assigned to the total energy function, W (or W_{PS}) for CbI and H_d for Standard PBC, at the desired equilibrium point. The proposition below shows that the use of a *scalar* dynamic extension in CbI, i.e., when we add only one integrator (equivalently, generate only one Casimir function), does not provide any additional freedom for minimum assignment to the corresponding static state–feedback solutions of Standard PBC.

Proposition 15: Consider the functions

$$
\begin{aligned}
W(x, \zeta) &\overset{\triangle}{=} H(x) + H_c(\zeta) + \Phi(\mathcal{C}(x) - \zeta) \\
H_d(x) &\overset{\triangle}{=} H(x) + H_c(\mathcal{C}(x)),
\end{aligned}
$$

with $\zeta \in \mathbb{R}$ and $\mathcal{C} : \mathbb{R}^n \to \mathbb{R}$. Then

$$
\begin{aligned}
\nabla W_\star = 0 \;\; &\text{and} \;\; \nabla^2 W_\star > 0 \\
&\Rightarrow (\nabla H_d)_\star = 0 \;\; \text{and} \;\; (\nabla^2 H_d)_\star > 0.
\end{aligned}
$$

Proof: Compute

$$\nabla W = \begin{bmatrix} \nabla H + \Phi' \nabla \mathcal{C} \\ H_c' - \Phi' \end{bmatrix}, \; \nabla H_d = \nabla H + H_c' \nabla \mathcal{C},$$

where $(\cdot)'$ denotes differentiation of a function of a scalar argument. Now,

$$\nabla W_\star = 0 \;\Rightarrow\; \Phi'_\star = (H_c')_\star \;\Rightarrow\; \nabla_x W_\star = (\nabla H_d)_\star = 0.$$

On the other hand,

$$
\begin{aligned}
\nabla^2 H_d &= \nabla^2 H + H_c' \nabla^2 \mathcal{C} + H_c'' \nabla \mathcal{C} \nabla \mathcal{C}^\top \\
\nabla^2 W &= \begin{bmatrix} \nabla^2 H + \Phi' \nabla^2 \mathcal{C} & 0 \\ 0 & H_c'' \end{bmatrix} + \\
&= + \Phi'' \begin{bmatrix} \nabla \mathcal{C} \nabla \mathcal{C}^\top & -\nabla \mathcal{C} \\ -\nabla \mathcal{C}^\top & 1 \end{bmatrix}
\end{aligned}
$$

Now,

$$
\begin{bmatrix} I & \nabla \mathcal{C} \\ 0 & |\nabla \mathcal{C}|^2 \end{bmatrix} \nabla^2 W \begin{bmatrix} I & \nabla \mathcal{C} \\ 0 & |\nabla \mathcal{C}|^2 \end{bmatrix}^\top =
\begin{bmatrix} \nabla^2 H + \Phi' \nabla^2 \mathcal{C} + H_c'' \nabla \mathcal{C} \nabla \mathcal{C}^\top & * \\ * & * \end{bmatrix}
$$

From Sylvester's Law of Inertia we have that $\nabla^2 W$ and the right hand side matrix above have the same inertia. Consequently,

$$
\begin{aligned}
\nabla^2 W_\star > 0 \;\Rightarrow\; &(\nabla^2 H + \Phi' \nabla^2 \mathcal{C} + H_c'' \nabla \mathcal{C} \nabla \mathcal{C}^\top)_\star > 0 \\
&\Leftrightarrow (\nabla^2 H_d)_\star > 0,
\end{aligned}
$$
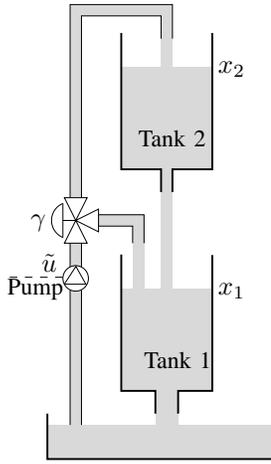
Fig. 3.   The two-tank system

where we used the fact that $\Phi'_\star = (H'_c)_\star$ in the last equivalence. ∎

Remark 24: Proposition 15 proves that if W has a stationary point at $(x_\star, \zeta_\star)$ and it is locally strictly convex around this point, then the same is true for H_d—with respect to $x_\star$.

VIII. EXAMPLES

A. Two–Tanks Level Regulation Problem

Consider the two–tank system depicted in Fig 3 with an input flow split between the tanks via a valve. The state variables $x_1 > 0$ and $x_2 > 0$ represent the water level in the lower and upper tank, respectively, and the control action $\tilde{u} \geq 0$ is the flow pumped from the reservoir. The valve parameter is the constant $\gamma \in [0, 1]$, with $\gamma = 0$ if the valve is fully open and $\gamma = 1$ if the valve is closed. We will assume in the sequel that $\gamma > 0$.

Using Torricelli's law the dynamics of the system can be written in PH form (1) with

$$\mathcal{J} = \begin{bmatrix} 0 & \alpha_2 \sqrt{x_2} \\ -\alpha_2 \sqrt{x_2} & 0 \end{bmatrix},$$

$$\mathcal{R} = \begin{bmatrix} \alpha_1 \sqrt{x_1} & 0 \\ 0 & 0 \end{bmatrix}, \quad g = \begin{bmatrix} 1 \\ g_2 \end{bmatrix}$$

(mass) energy function $H = x_1 + \frac{A_2}{A_1} x_2$, and cyclo–passive (constant) output $y = \frac{1}{\gamma}$. The system parameters are all positive and defined as

$$\alpha_i \triangleq \frac{a_i \sqrt{2G}}{A_i}, \; i = 1, 2, \quad g_2 \triangleq \frac{1 - \gamma}{\gamma},$$

where a_i, A_i are the cross–sections of the outlet holes and the tanks respectively, G is the gravitation constant, we defined $u \triangleq \frac{\gamma}{A_1} \tilde{u}$ and, to simplify notation, we assumed $A_1 = A_2$.[15] The achievable equilibrium set is the line

$$\mathcal{E} \triangleq \{\bar{x} \in \mathbb{R}^2_{>0} \mid \bar{x}_2 = \left[\frac{a_1}{a_2}(1 - \gamma)\right]^2 \bar{x}_1\},$$

[15]See [17], and references therein, for further details on the model.

and the control objective is to stabilize a given equilibrium point $x_\star \in \mathcal{E}$.

The dissipation obstacle hampers the application of CbI and EBC. Indeed, the condition (13) for CbI is not satisfied due to the presence of $\alpha_1 \sqrt{x_1}$ in the damping matrix $\mathcal{R}$ and the fact that the first coordinate has to be shaped. EBC is also not applicable because the control at the equilibrium $u_\star = \frac{\alpha_2}{g_2} \sqrt{x}_{2\star} \neq 0$—for all non–trivial points—hence, the power extracted at the equilibrium $u_\star y_\star \neq 0$.

We now consider Basic CbI_{PS} and start by investigating the condition for generation of new cyclo–passive outputs (23). This yields $\mathcal{R}F^{-1}g = \begin{bmatrix} \frac{-g_2 \alpha_1}{\alpha_2} \sqrt{\frac{x_1}{x_2}} \\ 0 \end{bmatrix} \neq 0$, hence, $y_{\mathsf{PS}} \neq y$. Unfortunately, the condition for existence of Casimirs for Basic CbI_{PS}, i.e., (24) with $F_d = F$, is not satisfied. Indeed, as can be easily verified, the vector

$$F^{-1}g = \begin{bmatrix} \frac{-g_2}{\alpha_2 \sqrt{x_2}} \\ \frac{1}{\alpha_2 \sqrt{x_2}} - \frac{g_2 \alpha_1}{\alpha_2^2} \frac{\sqrt{x_1}}{x_2} \end{bmatrix},$$

is not the gradient of a function.

The fact below, which ensures CbI_{PS} (with $F_d \neq F$) is applicable, can be verified via direct substitution.

Fact 1: The full rank constant matrix $F_d^{-1} = -\begin{bmatrix} a & a \\ 0 & d \end{bmatrix}$, with

$$a > 0, \quad 4d > a, \tag{53}$$

verifies $F_d \nabla H_{\mathsf{PS}} = F \nabla H$ and $F_d + F_d^\top < 0$, where

$$\nabla H_{\mathsf{PS}} = \begin{bmatrix} a\alpha_1 \sqrt{x_1} \\ d\alpha_2 \sqrt{x_2} \end{bmatrix}.$$

We compute the Casimirs for CbI_{PS} using (24), that we repeat here for ease of reference, $F_d \nabla \mathcal{C} = -g$. This yields

$$\mathcal{C}(x) = \frac{a}{\gamma} x_1 + dg_2 x_2.$$

The next step is to determine the functions H_c and Φ to assign the desired minimum to

$$W_{\mathsf{PS}}(x, \zeta) = H_{\mathsf{PS}}(x) + H_c(\zeta) + \Phi\left(\frac{a}{\gamma} x_1 + dg_2 x_2 - \zeta\right),$$

which, upon addition of a constant, will then qualify as a Lyapunov function for the controlled system. Some simple calculations prove that

$$\nabla W_{\mathsf{PS}}(x_\star, \zeta_\star) = 0 \quad \Leftrightarrow \quad \Phi'(\mathcal{C}(x_\star) - \zeta_\star) = -u_\star = H'_c(\zeta_\star).$$

The Hessian is given by

$$\nabla^2 W_{\mathsf{PS}} = \begin{bmatrix} \frac{a\alpha_1}{2} \frac{1}{\sqrt{x_1}} & 0 & 0 \\ 0 & \frac{d\alpha_2}{2} \frac{1}{\sqrt{x_2}} & 0 \\ 0 & 0 & H''_c \end{bmatrix} + \Phi'' \begin{bmatrix} \nabla \mathcal{C} \\ -1 \end{bmatrix} \begin{bmatrix} \nabla \mathcal{C}^\top & -1 \end{bmatrix}.$$

The first matrix in the right hand side is positive definite for all H_c such that $H''_c > 0$, while the second matrix is positive semi–definite provided $\Phi'' \geq 0$. This suggests the simple choice

$$H_c = \frac{\kappa}{2}\left(\zeta - \zeta_\star - \frac{1}{\kappa} u_\star\right)^2, \quad \Phi = -u_\star(\mathcal{C} - \zeta),$$

with $\kappa > 0$, which clearly satisfies the minimum conditions above.

The CbI_{PS} controller is obtained using (19) and setting $\dot{\zeta} = y_{\text{PS}}$, $u = -H'_c$ (with $v = 0$) to get the nonlinear dynamic state–feedback controller[16]

$$\Sigma_c + \Sigma_I^{\text{PS}} : \begin{cases} \dot{\zeta} = -\frac{a}{\gamma}\alpha_1\sqrt{x_1} + \left(\frac{a}{\gamma} - dg_2\right)\alpha_2\sqrt{x_2} + \\ \qquad + \left(\frac{a}{\gamma} + dg_2^2\right)[u_\star - \kappa(\zeta - \zeta_\star)] \\ u = u_\star - \kappa(\zeta - \zeta_\star), \end{cases}$$

where the free parameters a and d should satisfy (53), and κ is an arbitrary positive number. This controller ensures that, for any $x_\star \in \mathcal{E}$, and any $\zeta_\star \in \mathbb{R}$, $(x_\star, \zeta_\star)$ is a stable equilibrium of the closed–loop system with Lyapunov function $W_{\text{PS}}(x, \zeta) - W_{\text{PS}}(x_\star, \zeta_\star)$. See [30] for the modifications required to ensure asymptotic stability.

It is interesting to remark that, even though Basic $CbI_{\text{PS}}^{\text{SM}}$ incorporates more information about the plant, is not applicable for this problem. Indeed, although the Casimir function can be determined from $g^\perp F \nabla \mathcal{C} = 0$, it is easy to see that the Hessian $\nabla^2 W_{\text{PS}}$ is rank deficient, independently of the functions H_c and Φ.

We wrap–up this example showing that we can considerably simplify the design, restricting the dynamics of CbI_{PS} to the set $\zeta = \mathcal{C}(x)$ to obtain IDA–PBC—as suggested in Subsection VII-B. Towards this end, we set

$$\hat{u}_{\text{IDA}}(x) = -\nabla_{\mathcal{C}} H_c(\mathcal{C}(x)), \quad H_a(x) = H_c(\mathcal{C}(x)),$$

$$H_d(x) = H_{\text{PS}}(x) + H_a(x).$$

The simplest choice $H_c = -u_\star \mathcal{C}$ ensures $x_\star = \arg\min H_d(x)$ and yields the constant open loop control $\hat{u}_{\text{IDA}} = u_\star$. A more interesting option is to select

$$H_c = \frac{\kappa}{2}(\mathcal{C} - \mathcal{C}_\star)^2 - u_\star(\mathcal{C} - \mathcal{C}_\star), \quad \kappa > 0$$

which ensures that the linear controller

$$\hat{u}_{\text{IDA}} = \begin{bmatrix} k_1 & k_2 \end{bmatrix}(x - x_\star) + u_\star,$$

guarantees asymptotic stability of $x_\star$ for all $k_1 < 0$ and $4k_2 < g_2\gamma k_1$. This controller was derived, following the classical IDA–PBC methodology in the interesting paper [17].

B. A Nonlinear RC Circuit

Consider the circuit depicted in Fig. 4 consisting of a linear resistor, a nonlinear capacitor and a voltage source u. The capacitor is described by its electric energy function $H(x)$, with x the charge, and the constitutive relations $\dot{x} = i$ and $H' = v$, where v and i are the capacitors voltage and current, respectively. One way to represent the system in PH form is selecting

$$\mathcal{J} = 0, \quad \mathcal{R} = \frac{1}{R}, \quad g = \frac{1}{R}.$$

---

[16]In spite of its apparent complexity the control has a simple implementation, namely,

$$u = \frac{1}{\tau s + 1}[d_1\sqrt{x_1} + d_2\sqrt{x_2} + d_3(u_\star + \kappa\zeta_\star)] + u_\star + \kappa\zeta_\star,$$

with the constants τ, d_i taking values on some suitably defined ranges. This can, of course, be further simplified selecting $\zeta_\star = 0$.
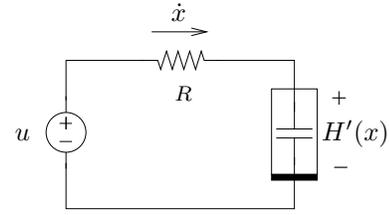
Fig. 4. Nonlinear RC circuit.
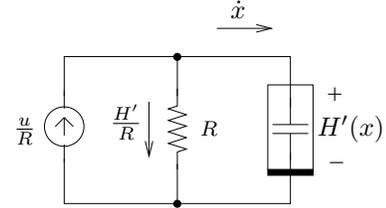
Fig. 5. Norton equivalent of the nonlinear RC circuit.

In this case, the port variables are the voltage u and the passive output $y = \frac{1}{R}H'$, while the power balance equation becomes

$$\dot{H} = -\frac{1}{R}(H')^2 + H'\frac{1}{R}u.$$

Even though, as expected, all the terms in this equation have units of power, the physical interpretation of this realization is best understood appealing to the Thevenin—Norton transformed systems represented in Fig. 5. The control objective is, again, to stabilize a given equilibrium point $x_\star \in \mathbb{R}$.

It is straightforward to see that there is no $\mathcal{C}$ that satisfies the Casimir conditions: $-\frac{1}{R}\mathcal{C}' = \frac{1}{R}$, $\frac{1}{R}\mathcal{C}' = 0$, so this problem can not be solved using CbI.

For any desired equilibrium point $x_\star$, the corresponding input $u_\star = (H')_\star$ and output $y_\star = \frac{1}{R}(H')_\star$ yield the power $u_\star y_\star = \frac{1}{R}(H')_\star^2$. Hence, the condition $u_\star y_\star = 0$, necessary for EB–PBC, can only be satisfied if the capacitor voltage at the equilibrium is zero. The fact that $\mathcal{R}F^{-1}g = -\frac{1}{R} \neq 0$ implies that $y \neq y_{\text{PS}}$, so we will try the CbI_{PS} technique.

It is worth mentioning that the general CbI_{PS}, where we look for F_d, H_d satisfying (16) and (17), yields new storage functions of the form $\frac{R_d}{R}H$, $R_d > 0$. Since scaling the energy function does not change its minima, we stay with Basic CbI_{PS}, i.e. $F_d = F$. The new system is thus

$$\Sigma_{(u, y_{\text{PS}})} : \begin{cases} \dot{x} = \frac{1}{R}(-H' + u) \\ y_{\text{PS}} = \frac{1}{R}(-H' + u), \end{cases} \tag{54}$$

where $H_{\text{PS}} = H$. We bring to the readers attention the fact that (54) is an alternative, actually more natural, realization of the system of Fig. 4, picking up as port variables voltage (u) and current ($\dot{x}$), which is equal to y_{PS}. The Casimirs for CbI_{PS} are solutions of $F\mathcal{C}' = -g$, which in this example becomes $\frac{1}{R}\mathcal{C}' = \frac{1}{R}$, so

$$\mathcal{C}(x) = x$$

will be a Casimir. We look now for functions H_c, Φ such that the function

$$W_{\text{PS}}(x, \zeta) = H(x) + H_c(\zeta) + \Phi(x - \zeta)$$
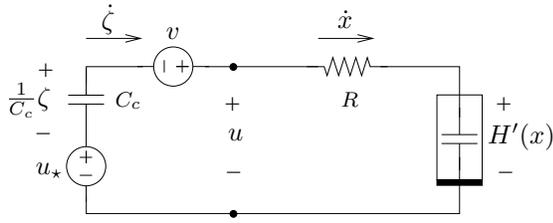
Fig. 6.   Nonlinear RC circuit with controller.

has an isolated minimum at a given equilibrium point $(x_\star, 0)$—where, for simplicity, we have taken $\zeta_\star = 0$. We have

$$(\nabla W_{\mathrm{PS}})_\star = 0 \quad \Leftrightarrow \quad \begin{cases} \Phi'_\star = -u_\star \\ (H'_c)_\star = -u_\star \end{cases}. \tag{55}$$

Some simple computations show that the Hessian $\nabla^2 W_{\mathrm{PS}} > 0$ if and only if

$$\Phi'' > -H'', \quad H''_c > \frac{-H'' \Phi''}{H'' + \Phi''}. \tag{56}$$

For the sake of simplicity, let us fix again a quadratic

$$H_c = \frac{1}{2C_c}(\zeta - C_c u_\star)^2,$$

with $C_c \in \mathbb{R}_+$, which satisfies the second condition of (55). For Φ we propose the second order polynomial $\Phi(z) = \frac{\beta}{2}z^2 + \gamma z$, where β and γ are constants to be defined. The first condition of (55) imposes the following constraint to the free parameters: $\beta x_\star + \gamma = -u_\star$. Evaluating conditions (56) at the equilibrium turns into

$$\beta > -H''_\star, \quad \frac{1}{C_c} > -\frac{H''_\star \beta}{H''_\star + \beta},$$

from where it is easy to see that if $H''_\star > 0$, we can take $\beta = 0$ and the equilibrium $(x_\star, 0)$ will be stable, for all $C_c > 0$, with Lyapunov function $W_{\mathrm{PS}}(x, \zeta) - (W_{\mathrm{PS}})_\star$, where

$$W_{\mathrm{PS}}(x, \zeta) = H(x) + \frac{1}{2C_c}(\zeta - C_c u_\star)^2 - u_\star(x - \zeta).$$

The controller is given by

$$\Sigma_c + \Sigma_I^{\mathrm{PS}} : \begin{cases} \dot{\zeta} = \frac{1}{R}(-H' + u_\star - \frac{1}{C_c}\zeta + v) \\ u = u_\star - \frac{1}{C_c}\zeta + v. \end{cases}$$

As shown in Fig. 6, it has a physical interpretation as a capacitor with charge ζ and capacitance C_c in series with a constant voltage source $u_\star$, coupled with the system of Fig. 4.

Before wrapping–up this example let us illustrate with it the relation between Brayton–Moser and PH models briefly discussed in Subsection VI-D and thoroughly explained in [33]. To transform from one to the other we assume the function $v = H'(x)$, is invertible. That is, there exists a function $\hat{x}(v)$ such that $H'(\hat{x}(v)) = v$. Define the Legendre transform $\tilde{H}(v) \triangleq v\hat{x}(v) - H(\hat{x}(v))$. Differentiating the latter with respect to v and evaluating at $(\hat{x}(v), v)$, it easy to see that $x = \tilde{H}'(v)$.[17] Differentiating x with respect to time we

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[17]In the case of a linear capacitor, $H(x) = \frac{1}{2C}x^2$, $\hat{x}(v) = Cv$, and $\tilde{H}(v) = \frac{C}{2}v^2$.

get

$$\tilde{H}'' \dot{v} = \frac{1}{R}(-v + u),$$

which is in the Brayton–Moser form (47) with $Q(v) = -\tilde{H}''(v)$ the, so–called, generalized capacitance, $B = -1$, input the current $\frac{1}{R}u$ and mixed potential, $P(v) = \frac{1}{2R}v^2$, the power dissipated in the resistor. See Fig. 5.

Multiplying by $\dot{v}$, and assuming that $\tilde{H}'' \geq 0$, we obtain the cyclo–passivity inequality

$$\dot{P} \leq \frac{1}{R}u\dot{v}.$$

It is interesting to note that the characterization of electrical circuits that verify this kind of cyclo–passivity inequalities (or the dual $\dot{\tilde{P}} \leq v\dot{u}$) is an essential step in the solution of the power factor compensation problem of energy transformation systems [10].

IX. CONCLUDING REMARKS AND FUTURE RESEARCH

We have investigated in this paper the relationships between CbI and Standard PBC. We have concentrated our attention on the ability of the methods to shape the energy function and the role of dissipation to fulfill this task. Energy–shaping is, of course, the key step for the successful application of PBCs and, similarly to all existing methods for nonlinear systems controller (or observer) design, requires the solution of a set of PDEs. In the case of CbI the solutions of the PDEs are the Casimir functions $\mathcal{C}$ and, eventually, F_d. On the other hand, for Standard PBC their solution directly provides the "added" energy function H_a, with F_d a free parameter for IDA–PBC or a solution of another PDE for PS–PBC. The various methods have been classified comparing the size of the solution sets of these PDEs.

To enlarge the domain of application of CbI several variations of the method have been considered—all of them considering the simple (m–th order) nonlinear integrator controller subsystem Σ_c given in (6). Also, various popular Standard PBCs have been derived adopting a unified perspective, i.e., fixing the desired dissipation and writing a linear PDE for the unknown added energy function.

There are many open question and topics for further investigation including:

i) It is well known [29], that the flexibility provided by the free parameter F_d in IDA–PBC is essential to solve many practical problems. As seen from the diagram of Fig. 2 there is no CbI version of IDA–PBC. What is the modification to CbI that is needed to add this degree of freedom?

ii) As indicated in Remark 23 PS–PBC (or equivalently $CbI_{\mathrm{PS}}^{\mathrm{SM}}$) suggests a two–step procedure to solve the non–homogeneous PDE of IDA–PBC. Instead of fixing F_d and solving the PDE for H_d as is sometimes done in IDA–PBC, it is proposed to find F_d as a (suitable) solution of the new PDE (18). This procedure does not generate all solutions of (46). However, given the intrinsic difficulty of defining a "suitable" F_d that will simplify (46), it is interesting to explore the decomposition as an alternative for generation of F_d. In this respect,

the parametrization of the solutions given in Remark 10 is of particular importance.

iii) For ease of presentation we have fixed the order of the dynamic extension to be m. However, as indicated in Remark 7 there are some advantages for increasing their number. Also, for simplicity we have taken simple nonlinear integrators, further investigations are required to see if other structures could be of use.

iv) We have concentrated our attention on the ability of the various PBCs to modify the energy function, without particular concern to stabilization. In particular, we have only briefly addressed in Remark 6 the issue of asymptotic stabilization, that arises naturally in CbI where the sets Ω_κ are rendered invariant. Imposing a constraint on the controller initial conditions is, of course, not practically reasonable, and is suggested there only to illustrate the problem. In [30] we propose two alternative solutions: an adaptive scheme that "estimates" $\zeta_\star$, and the addition of damping to the controller.

v) Proposition 15 shows that, in the single input case, the use of a dynamic extension does not provide any additional freedom for minimum assignment to the corresponding static state–feedback solutions. On the other hand, the use of dynamic extension certainly has an impact on performance and might provide simpler controller expressions. Assessment of the performance improvement (or degradation) is a difficult task that will be investigated in the future.

vi) A special class of PBC has been successfully derived for systems described by Euler–Lagrange equations of motion—which includes, among others, mechanical, electromechanical and power electronic systems—see [23] for a summary of the main results. The key structural property of these systems that is exploited in the controller design is the presence of work–less forces, that is well–known in mechanics [26] and captured via the skew–symmetry of the matrix $\dot{M}(q) - 2C(q,\dot{q})$, where M is the inertia matrix and $C\dot{q}$ are the Coriolis and centrifugal forces. This strong property, which is independent of passivity of Euler–Lagrange (or PH) systems [34], has not been used in CbI or Standard PBC and it would, certainly, be interesting to incorporate it in these designs.

vii) The procedure to generate new cyclo–passive outputs of Section III is of interest independently of its application to CbI. Indeed, several control problems can be recast in terms of identification of "suitable" (cyclo–)passive outputs, which are known to be easy to be regulated—for instance with a simple PI law. Two practical applications where this idea has been applied are reported in [12], [31].

viii) As explained in Subsection VI-D our research on power shaping was motivated by the study of Brayton–Moser models of nonlinear RLC circuits, for which the solution of the critical PDE (18) is simplified. It is interesting to explore modelling procedures for other classes of physical systems, e.g., mechanical systems, that will yield this kind of structures. See [14] for some results along this direction.

APPENDIX

TABLE I
LIST OF ACRONYMS

BIDA	Basic IDA
CbI	Control by interconnection
CbI^{SM}	Control with state modulated interconnection
CbI_{PS}	CBI with power shaping output
$CbI_{\mathrm{PS}}^{\mathrm{SM}}$	Control with power shaping output and state modulated interconnection
EB	Energy–balancing
IDA	Interconnection and damping assignment
PBC	Passivity–based control
PDE	Partial differential equation
PS	Power shaping
y_{PS}	Power shaping output

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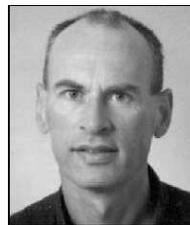
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