Lecture 22

Adaptive SMC

The main obstacles for application of Sliding Mode Control are two interconnected phenomena:

- chattering,
- and high activity of control action.

Claim 22.1 *As we can see from the previous considerations that the amplitude of chattering is proportional to the magnitude of a discontinuous control.*

This two problems can be handled simultaneously if the magnitude is reduced to a minimal admissible level defined by the conditions for sliding mode to exist. The adaptation methodology for obtaining the minimum possible value of control is based on two approaches developed in recent publications [15]:

- *The σ - adaptation*, providing the adequate adjustments of the magnitude of a discontinuous control within the "reaching phase", that is, when state trajectories are out of a sliding surface [16];
- *Dynamic adaptation* or the adaptation within a sliding mode (on a sliding surface), based on the, so-called, *Equivalent Control Method* (ECM) obtained by the direct measurements of the output signals of a first-order low-pass filter containing in the input the discontinuous control with the specially adapted magnitude value [17].

22.1 The σ-adaptation method

Consider the nonlinear uncertain system

$$\left.\begin{aligned} \dot{x}(t) &= f(x(t)) + g(x(t))\,u, \\ x(0) &= x_0,\ \ t \geq 0 \end{aligned}\right\} \tag{22.1}$$

where $x(t) \in \mathcal{X} \subset \mathbb{R}^n$ the state vector and $u \in \mathbb{R}$ the control input to be designed. The function $f(x)$ and the vector function $g(x)$ are supposed to be smooth uncertain functions and are bounded for all $x \in \mathcal{X}$; furthermore, $f(x)$ contains unmeasured perturbations term and $g(x) > 0$ for all $x \in \mathcal{X}$.

The control objective consists in forcing the continuous function $\sigma(x,t)$, named *sliding variable*, to 0. Supposing that σ admits the relative degree equal to 1 with respect to u, one gets

$$\left.\begin{aligned} \dot{\sigma}(x,t) &= \psi(x,t) + \gamma(x,t)\,u, \\ \psi(x,t) &:= \frac{\partial\sigma(x,t)}{\partial t} + \left(\frac{\partial\sigma(x,t)}{\partial x}\right)^{\mathsf{T}} f(x), \\ \gamma(x,t) &:= \left(\frac{\partial\sigma(x,t)}{\partial x}\right)^{\mathsf{T}} g(x) \end{aligned}\right\} \tag{22.2}$$

Functions $\psi(x,t)$ and $\gamma(x,t)$ are supposed to be bounded such that for all $x \in \mathcal{X}$ and all $t \geq 0$

$$|\psi(x,t)| \leq \psi_M,\ \ 0 < \gamma_m \leq \gamma(x,t) \leq \gamma_M \tag{22.3}$$

It is assumed that ψ_M, γ_m and γ_M exist but are *not known*. The objective for a designer is to propose a sliding mode controller $u(\sigma,t)$ with the same features as classical SMC, namely, robustness and finite-time convergence but without any information on uncertainties and perturbations (appearing in $f(x)$). Furthermore, this objective allows to ensure a global stability of closed-loop system whereas the classical way (with knowledge of uncertainties bounds) only ensures its semi-global stability.

In the sequel, the definitions of *ideal* and *real* sliding mode are recalled.

Definition 22.1 *We say that an* ***ideal sliding mode*** *exists if*

$$\lim_{\sigma \to +0} \dot{\sigma} < 0 \ \textit{and} \ \lim_{\sigma \to -0} \dot{\sigma} > 0.$$

If, due to some small positive parameter μ, the state trajectories belong to domain

$$|\sigma(t)| \le \Delta(\mu), \ \lim_{\mu\to 0} \Delta(\mu) = 0,$$

then the motion is called a ***real sliding mode****.*

In real applications, an "ideal" sliding mode, as defined in Definition 22.1, cannot be established. But the concept of a "real" sliding mode seems to be workable.

As it is common for Sliding Mode Theory, we will consider the scalar discontinuous control action $u = u(\sigma, t)$ at time t as

$$\left.\begin{array}{c} u(\sigma,t) = -k(t)\operatorname{sign}(\sigma(x(t),t)), \\ \dot{k}(t) = \begin{cases} \bar{k}\,|\sigma(x(t),t)|\operatorname{sign}(|\sigma(x(t),t)| - \varepsilon) & \text{if } k(t) > \mu \\ \\ 0 \text{ if } k(t) \le \mu \end{cases} \end{array}\right\} \tag{22.4}$$

with $\bar{k} > 0$, $\varepsilon > 0$ and a small enough positive μ. The parameter μ is introduced in order to get only positive values for $k(t)$.

Once sliding mode with respect to $\sigma(x(t), t)$ is established, the proposed gain-adaptation law (22.4) allows the gain $k(t)$ declining (while $|\sigma(x(t), t)| < \varepsilon$). In other words, the gain $k(t)$ will be kept at the smallest level that allows a given accuracy of σ - stabilization. Of course, as described in the sequel, this adaptation law allows to get an adequate gain with respect to uncertainties/perturbations magnitude.

Theorem 22.1 ([16]) *Given the nonlinear uncertain system (22.1) with the sliding variable $\sigma(x(t), t)$ dynamics (22.2) controlled by (22.4), there exists a finite time t_f so that a real sliding mode is established for all $t \ge t_f$, i.e., $|\sigma(x(t), t)| < \delta$ for all $t \ge t_f$ with*

$$\delta = \sqrt{\varepsilon^2 + \psi_M^2 / \left(\bar{k}\gamma_m\right)}. \tag{22.5}$$

So, the convergence to the domain $|\sigma(x(t), t)| \le \varepsilon$ is in a finite time, but could be sustained in the bigger domain $|\sigma(x(t), t)| \le \delta$. Therefore, the real sliding mode exists in the domain $|\sigma(x(t), t)| \le \delta$. The choice of parameter ε has to be made by an adequate way because a 'bad' tuning could provide either instability and control gain increasing to infinity, or bad accuracy for closed-loop system. In [16] there is suggested to select ε adjusted in time as

$$\varepsilon(t) = 4k(t)t_f.$$

22.2 The dynamic adaptation based on ECM

22.2.1 The simple motivating example

Consider the **first-order scalar system**

$$\left.\begin{aligned} \dot{x}(t) &= a(t) + u, \\ u &= -k\,\mathrm{sign}(x(t)),\ k > 0. \end{aligned}\right\} \tag{22.6}$$

The ranges of a time varying parameter

$$0 < |a(t)| \le a_+$$

and the upper bound A for its time derivative

$$|\dot{a}(t)| \le A$$

are known only. The sliding mode with $x(t) \equiv 0$ exists for all values of unknown parameter $a(t)$ if $k > a_+$. However if parameter $a(t)$ is varying, the gain k can be decreased and, as a result, chattering amplitude can be reduced. The objective of adaptation is decreasing k to the minimal value preserving sliding mode, if parameter a is unknown. If the condition $k > a_+$ holds, then sliding mode with $x(t) \equiv 0$ occurs and control in (22.6) should be replaced by the, so-called, *equivalent control* u_{eq} (see [1] and lecture 17) for which the right-hand side in (22.6) is equal to zero, namely,

$$\dot{x}(t) = 0 = a(t) + u_{eq}, \tag{22.7}$$

that leads to

$$\left.\begin{gathered} k(t)\left[\mathrm{sign}(x(t))\right]_{eq} = a(t) \\ \Downarrow \\ k(t)\left|\left[\mathrm{sign}(x(t))\right]_{eq}\right| = |a(t)| \end{gathered}\right\} \tag{22.8}$$

If $k < |a(t)|$, the set $x(t) \equiv 0$ is of zero measure in time and can be disregarded. The function $[\mathrm{sign}(x(t))]_{eq}$ is an average value, or a slow component of discontinuous function $\mathrm{sign}(x(t))$ switching at high frequency and can be easily obtained by a *low pass filter filtering* out the high frequency component. Of course, the average value is in the range $(-1, 1)$. Then the design idea of adaptation seems to be evident:

after sliding mode occurs the control parameter $[\mathrm{sign}(x(t))]_{eq}$ *should be increased until it becomes close to* 1.

On one hand, the condition $k(t) > a(t)$ should hold. But the chattering amplitude is proportional to $k(t)$. The objective of adaptation process looks now transparent:

the gain $k(t)$ should tend to $|a(t)|/\alpha$ with $\alpha \in (0,1)$ which is very close to 1.

As a result, the minimal possible value of discontinuity magnitude is found for the current value of parameter $a(t)$ to reduce the amplitude of chattering. For that purpose select the *adaptation algorithm* in the form

$$\left.\begin{array}{c} \dot{k}(t) = \rho k(t)\text{sign}(\delta(t)) - M[k(t) - k^{+}]_{+} + M[\mu - k(t)]_{+}, \\ \delta(t) := \left|[\text{sign}(x(t))]_{eq}\right| - \alpha,\ \alpha \in (0,1), \\ [z]_{+} := \left\{\begin{array}{lll} 1 & \text{if} & z \geq 0 \\ 0 & \text{if} & z < 0 \end{array}\right.,\ M > \rho k^{+},\ k^{+} > a^{+},\ \rho > 0. \end{array}\right\} \quad (22.9)$$

The gain k can vary in the range $[\mu, k^{+}]$, $\mu > 0$ is a preselected minimal value of k. For the adaptation algorithm (22.9) sliding mode will occur after a finite time interval. Indeed, if it does not exist, then

$$\left|[\text{sign}(x(t))]_{eq}\right| = 1,$$

that leads to $\delta > 0$, and the increasing gain $k(t)$ will reach the value k^{+} which is sufficient for enforcing sliding mode for any value of parameter $a(t)$. In sliding mode the adaptation process (22.9) with $\delta(t) = 0$ is over after a finite time t_f. Indeed, calculate the time derivative of the Lyapunov function $V(\delta) = \delta^2/2$ assuming that during the adaptation process $k(t) \in [\mu, k^{+}]$ which means that $|a(t)|/\alpha > \mu$ or ($|a(t)| > \alpha\mu$). The time derivatives of $\left|[\text{sign}(x(t))]_{eq}\right|$ (22.8) and $|a(t)|$ exist and the terms depending on M in the adaptation algorithm (22.9) are equal to zero. Calculate the time derivative of the Lyapunov function $V(\delta)$ by virtue of (22.8) and (22.9):

$$\left.\begin{array}{l} \dot{V}(\delta) = \delta\dot{\delta} = \delta\dfrac{d}{dt}\left|[\text{sign}(x)]_{eq}\right| = \delta\dfrac{d}{dt}(|a|/k) = \\ -|a|\,\delta k^{-1}\rho\text{sign}\left(\delta - M[k - k^{+}]_{+} + M[\mu - k]_{+}\right) \\ +\ \delta k^{-1}\dot{a}\text{sign}(a) \leq -|a|\,\delta k^{-1}\rho\text{sign}(\delta) + |\delta|\,k^{-1}A \\ \leq -\alpha\mu\rho k^{-1}|\delta| + |\delta|\,k^{-1}A = -|\delta|\,k^{-1}(\alpha\mu\rho - A) \end{array}\right\} \quad (22.10)$$

and if $\rho > A/\alpha\mu$ it follows

$$\dot{V}(\delta) \leq -\sqrt{2}\frac{(\alpha\mu\rho - A)}{k^+}\sqrt{V(\delta)},$$

implying that $\sqrt{V(\delta)} = 0$ at least after

$$t_f = \frac{k^+}{(\alpha\mu\rho - A)}\sqrt{2V(\delta(0))} = \frac{k^+}{(\alpha\mu\rho - A)}|\delta(0)|$$

and, as a result, $\delta(t)$ becomes equal to zero identically after the finite time t_f.

After the adaptation process is over ($t > t_f$) we have

$$\left|[\operatorname{sign}(x(t))]_{eq}\right| = \frac{|a|}{k} = \alpha.$$

So, $k = |a|/\alpha$. If in the course of motion $|a(t)|/\alpha < \mu$, then the gain $k(t)$ decreases until $k(t) = \mu$ and, as it follows from (22.9), it will be maintained at this level. Since the gain $a(t)$ is time varying its increase can result in $|a(t)|/\alpha = \mu$ and $\delta(t) = 0$ at a time t_f. As it follows from the above analysis, for the further motion in the domain $k(t) \in (\mu, k^+]$ with the initial condition $\delta(t_f) = 0$ the time function $\delta(t)$ will be equal to zero with $k(t) = |a(t)|/\alpha$.

Remark 22.1 *The function* $[\operatorname{sign}(x(t))]_{eq}$ *is needed here for the implementation of the adaptation algorithm (22.9). It can be derived by filtering out a high frequency component of the discontinuous function* $\operatorname{sign}(x(t))$ *by a low pass filter*

$$\tau\dot{z} + z = \operatorname{sign}(x(t)),\ z(0) = 0$$

with a small time constant $\tau > 0$ *and the output* $z(t)$ *which is, in fact, an estimate of* $[\operatorname{sign}(x(t))]_{eq}$ *satisfying*

$$\left|z(t) - [\operatorname{sign}(x(t))]_{eq}\right| \leq O(\tau) \underset{\tau\to 0}{\to} 0.$$

Then the convergence analysis of (22.9)-(22.10) with $\delta(t) = z(t) - \alpha$ *is valid beyond the domain* $|\delta(t)| \leq O(\tau)$*. This inequality defines the accuracy of adaptation. Note that the switching frequencies of the modern power converters are of order dozens of kHz, and very small time constant* τ *can be selected to get a high accuracy of adaptation.*

Notice also that, as follows from [1],

$$z(t) = \psi(t) + O(\sup|x\,(t)\,| + \tau) + O(\sup|x\,(t)\,|/\tau)$$

where $\psi(t)$ is the fast rate exponentially decreasing function. The term $\sup|x\,(t)\,|$ is inverse proportional to the sliding mode frequency f. It is of order of dozen kHz in the modern switching devices. Therefore it is not a problem to make the term

$$O(\sup|x| + \tau) + O(\sup|x|/\backslash\tau)$$

negligible. Of course, this engineering language can be translated into mathematical one, for example as follows: *for any* $\varepsilon > 0$ *there exists a switching frequency* f_0 *such that* $|z - u_{eq}| < \varepsilon$ if $f > f_0$ implying

$$\mathrm{sign}[\left|[\mathrm{sign}\,(x\,(t))]_{eq}\right| - \alpha] = \mathrm{sign}[|z\,(t)| - \alpha]$$

22.2.2 Multidimesiona case

Main assumptions

Here we consider an arbitrary order system

$$\boxed{\left.\begin{array}{c} \dot{x}\,(t) = f\,(t, x\,(t)) + b\,(t, x\,(t))\,u\,(t, x\,(t))\,, \\ \\ x\,(t) \in \mathbb{R}^n,\ f : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n, \\ \\ u : R^+ \times \mathbb{R}^n \to \mathbb{R},\ b : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n \end{array}\right\}} \tag{22.11}$$

for which we assume that

A1 the control $u = u\,(t, x)$ enforces siding mode on some surface

$$\sigma\,(x) = 0\,(\sigma \in C^1)$$

and is in the following form

$$\boxed{\left.\begin{array}{c} u\,(t, x) = -k\,(t)\left(1 + \lambda\sqrt{\|x\|^2 + \varepsilon}\right)\mathrm{sign}\,(\sigma\,(x))\,, \\ \\ \lambda \geq 0,\ \varepsilon > 0,\ k\,(t) \in [\mu, k^+]\,,\ \mu > 0. \end{array}\right\}} \tag{22.12}$$

Similarly to the example (22.6) the control gain $k\,(t)$ is a time varying function governed by the adaptation procedure described below.

A2 the uncertain functions $f(t,x)$ and $b(t,x)$ satisfy the commonly accepted conditions (which are much more general then in (22.3)):

$$\left.\begin{aligned} \|f(t,x)\| &\le f_0 + f_1\|x\|, \\ 0 < b_0 &\le \nabla^{\intercal}\sigma(x)\, b(t,x), \\ \|b(t,x)\| \le b^{+},\ &\|\nabla\sigma(x)\| \le \sigma^{+}, \end{aligned}\right\} \tag{22.13}$$

$$\left.\begin{aligned} \Phi(t,x) &:= \frac{\nabla^{\intercal}\sigma(x)\, f(t,x)}{\nabla^{\intercal}\sigma(x)\, b(t,x)}, \\ \|\nabla^{\intercal}\Phi(t,x)\| &\le \Phi_0 + \Phi_1\|x\|, \\ \left|\frac{\partial}{\partial t}\Phi(t,x)\right| &\le \varphi_0 + \varphi_1\|x\|. \end{aligned}\right\} \tag{22.14}$$

All coefficients in the right-hand sides of these inequalities are constant and positive. The function $\sigma(x)$ and its time derivative

$$\left.\begin{aligned} \dot{\sigma}(x) = \nabla^{\intercal}\sigma(x)\, f(t,x) - \\ \nabla^{\intercal}\sigma(x)\, b(t,x)\, k(t)\left(1+\lambda\sqrt{\|x\|^2+\varepsilon}\right)\operatorname{sign}(\sigma(x)) \end{aligned}\right\} \tag{22.15}$$

should have opposite signs ($\sigma(x)\dot{\sigma}(x) < 0$ if $\sigma(x) \neq 0$) for sliding mode to exist on the surface $\sigma(x) = 0$. The sufficient condition for this follows from (22.13),(22.14) and (22.15):

$$\begin{gathered} \sigma(x)\dot{\sigma}(x) = \sigma(x)\nabla^{\intercal}\sigma(x)\, f(t,x) - \\ \nabla^{\intercal}\sigma(x)\, b(t,x)\, k(t)\left(1+\lambda\sqrt{\|x\|^2+\varepsilon}\right)|\sigma(x)| \\ \le [\nabla^{\intercal}\sigma(x)\, b(t,x)]\, |\sigma(x)| \times \\ \left(|\Phi(t,x)| - k(t)\left(1+\lambda\sqrt{\|x\|^2+\varepsilon}\right)\right) < 0, \end{gathered}$$

if

$$|\Phi(t,x)| - k(t)\left(1+\lambda\sqrt{\|x\|^2+\varepsilon}\right) < 0, \tag{22.16}$$

which is always holds when

$$\lambda \geq f_1/f_0,\ \mu > f_0\sigma^+/b_0,\ k(t) \in \left(\mu, k^+\right] \tag{22.17}$$

in view of the relation

$$|\Phi(t,x)| - k(t)\left(1 + \lambda\sqrt{\|x\|^2 + \varepsilon}\right) \leq$$

$$f_0\frac{\sigma^+ (1 + \|x\| f_1/f_0)}{b_0} - \mu(1 + \lambda\|x\|).$$

To derive the sliding mode equation the function $\text{sign}(\sigma(x))$ should be replaced by the solution of the equation $\dot{\sigma}(x) = 0$ with respect to the term $\text{sign}(\sigma(x))$, called *the equivalent control*:

$$[\text{sign}(\sigma(x))]_{eq} := \begin{cases} \dfrac{\Phi(t,x)}{k(t)\left(1 + \lambda\sqrt{\|x\|^2 + \varepsilon}\right)} & \begin{array}{c}\text{if} \\ \sigma(x(t)) = 0\end{array} \\ \text{sign}(\sigma(x(t))) & \begin{array}{c}\text{if} \\ \sigma(x(t)) \neq 0\end{array} \end{cases} \tag{22.18}$$

satisfying (in view of (22.16)) in the sliding mode ($\sigma(x(t)) = 0$)

$$\left|[\text{sign}(\sigma(x))]_{eq}\right| < 1 \tag{22.19}$$

Description of the adaptation procedure The idea of the *adaptation law* for the control gain $k(t)$ is similar to that for our first-order system in the previous subsection:

$$\boxed{\dot{k}(t) = \begin{cases} (\gamma_0 + \gamma_1\|x\|)\, k(t)\text{sign}(\delta(t)) \\ -\, M[k(t) - k^+]_+ + M[\mu - k(t)]_+, \end{cases}} \tag{22.20}$$

where

$$\left.\begin{array}{c} \delta(t) := \left|[\text{sign}(\sigma(x(t)))]_{eq}\right| - \alpha, \\ \alpha \in (0,1),\ \lambda > 0,\ \gamma_0, \gamma_1 > 0. \end{array}\right\} \tag{22.21}$$

Select in (22.20)

$$k^+ > \sigma^+\frac{f_0}{b_0}.$$

If sliding mode does not exist, then

$$\left|[\mathrm{sign}\,(\sigma\,(x))]_{eq}\right| = 1,$$

and the gain $k(t)$ will be equal to k^+ which results in the occurrence of this motion in the surface $\sigma\,(x\,(t)) = 0$.

Theorem 22.2 ([17]) *For the dynamic system (22.11) closed by the control (22.12) with the gain adaptation law (22.20) - (22.21) with the parameters satisfying*

$$\boxed{\left.\begin{array}{c} k^+ > \sigma^+ \dfrac{f_0}{b_0},\ \ \mu > f_0\sigma^+/b_0,\ \ 0 < \varepsilon \ll 1, \\[2ex] \gamma_0 > \alpha^{-1}\left[\left(\dfrac{f_0}{\mu}+b^+\right)\Phi_0+\dfrac{\varphi_0}{\mu}+f_0+b^+k^+\right], \\[2ex] \gamma_1 \geq \alpha^{-1}\left(\dfrac{f_0}{\mu}+b^+\right)\Phi_1,\ \ M > \gamma_0 k^+, \end{array}\right\}} \tag{22.22}$$

there exist

$$\theta := \alpha\gamma_0 - \left[\left(\frac{f_0}{\mu}+b^+\right)\Phi_0 + \frac{\varphi_0}{\mu} + f_0 + b^+k^+\right] > 0 \tag{22.23}$$

and

$$t_f = \theta^{-1}\left|\delta\,(0)\right|$$

(where $\delta\,(0)$ is defined by (22.21)) such that for all $t \geq t_f$ the condition

$$\left|[\mathrm{sign}\,(\sigma\,(x\,(t)))]_{eq}\right| = \alpha \tag{22.24}$$

holds. It means that the sliding surface $\sigma\,(x) = 0$ is attained in a finite time t_f, and for $\alpha = 1 - \varepsilon_0$ ($\varepsilon_0 > 0$) is a small enough positive number) the suggested adaptation procedure provides $k\,(t)$ tending to a vicinity of the minimum possible value $k_{\min}\,(t)$, that is, as it follows from (22.18), in sliding mode

$$\boxed{\left.\begin{array}{c} k(t) = \left\{\begin{array}{ccc} \dfrac{1}{1-\varepsilon_0}k_{\min}\,(t) & if & k_{\min}\,(t) \geq \mu \\ \mu & if & k_{\min}\,(t) < \mu \end{array}\right. , \\[3ex] k_{\min}\,(t) := \dfrac{\left|\Phi(t, x\,(t))\right|}{1+\lambda\sqrt{\left\|x\,(t)\right\|^2+\varepsilon}}. \end{array}\right\}} \tag{22.25}$$

22.2.3 Super-twist control with adaptation

The dynamic system with super-twist controller can be represented as

$$\begin{cases} \dot{x}_1 = x_2 - \bar{\alpha}\sqrt{|x_1|}\mathrm{sign}(x_1), \\ \\ \dot{x}_2 = \phi(t) - k\mathrm{sign}(x_1). \end{cases} \tag{22.26}$$

Take $\sigma(x) = x_1$ and permit for the gain parameter to be time-varying, i.e.,

$$k = k\left(t\right).$$

Theorem 22.3 (on adaptive super-twist [17]) *The system (22.26) with disturbances $\phi(t)$ having a bounded derivative (fulfilling $\frac{d}{dt}\left|\phi(t)\right| \leq L$), and with the parameter $k(t)$ adapted on-line according to the adaptation law*

$$\boxed{\begin{array}{c} \dot{k}\left(t\right) = \begin{cases} \gamma_0 k(t)\mathrm{sign}\left(\delta\left(t\right)\right) - M\left[k(t) - k^{+}\right]_{+} + M\left[\mu - k(t)\right]_{+} \\ \text{if } 0 < \mu \leq k\left(t\right) \leq k^{+}, \\ \\ 0 \text{ otherwise} \end{cases} \\ \delta(t) := \left|\left[\mathrm{sign}\left(\sigma\left(x\right)\right)\right]_{eq}\right| - \alpha,\ \alpha = 1 - \varepsilon_0,\ \gamma_0 > L/\mu \end{array}} \tag{22.27}$$

converges in the finite time

$$t_f = \left|\delta\left(0\right)\right| k^{+} / \left(\mu\gamma_0 - L\right)$$

to the sliding mode regime $\sigma(x) = x_1 = 0$ maintaining within the relation

$$\left|\phi(t)\right| / \left|k(t)\right| = \alpha = 1 - \varepsilon_0$$

for small enough $\varepsilon_0 > 0$.

22.3 Exercises

Exercise 22.1 *For the same system as in Exercise (17.1) compare the* ***adaptive SM controllers*** *based on σ-adaptation and the equivalent control method.*

Exercise 22.2 *For the super-twist controller*

$$\left.\begin{array}{c} \dot{x}_1 = x_2 - \bar{\alpha}\sqrt{|x_1|}\mathrm{sign}(x_1) \\ \dot{x}_2 = \phi(t) - k\,(t)\,\mathrm{sign}(x_1) \\ \\ \bar{\alpha} = 0.5,\ x_1\,(0) = 1,\ x_2\,(0) = -1 \\ \phi(t) = \phi(0)\sin(\omega t)\ \ \textit{is unmeasurable signal} \\ \phi(0) = 0.1,\ \omega = 2 \end{array}\right\}$$

design the adaptive gain parameter $k\,(t)$ *(22.27) providing* $x_1\,(t) \simeq 0$ *after* $t > t_f$*, and demonstrate the figure depicting* $k\,(t)$ *and* $|\phi(t)|$ *which should be very closed.*

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