Lecture 10

Robust State Feedback control

This lecture looks at a specific form of nonlinearly affine control system with a suitably broad range of uncertainty. Here nonlinear uncertain systems are governed by a vector Ordinary Differential Equation (ODE) with quasi-Lipschitz right-hand sides that accept a wide range of external and internal uncertainty (including discontinuous nonlinearities such as relay and hysteresis elements, time-delay blocks and so on). The linear state-feedback controllers are investigated. The sufficient criteria that guarantee the boundedness of all conceivable controlled system trajectories are provided. Because any limited dynamics may be imposed in an ellipsoid, it is proposed that the "robust-optimal" gain-matrices of the planned linear feedbacks be chosen in such a manner that the "size" of this appealing ellipsoid is kept to a minimum. Several numerical examples are used to illustrate the point.

10.1 Proportional linear feedback design

10.1.1 Model description

Here we consider the quasi-Lipschitz affine controlled system (9.18) presented in the following quasi-linear format:

$$\boxed{\left.\begin{array}{c} \dot{x}(t) = Ax(t) + Bu(t) + \xi(t,x),\ x(0) = x_0, \\ \\ y(t) = x(t) \end{array}\right\}} \tag{10.1}$$

with

$$\left.\begin{array}{c}\xi(t,x) := f\left(x,t\right) - Ax\left(t\right) + \xi_x\left(t\right) = \\ f\left(x,t\right) - Ax\left(t\right) + \bar{\xi}_x\left(t\right) + \left(BB^{+} - I\right)\varphi\left(x^{*}\left(t\right),t\right),\end{array}\right\} \tag{10.2}$$

where

$x\left(t\right) \in \mathbb{R}^n$ is the state vector at time $t \in \mathbb{R}_+ := \{t : t \geq 0\}$,

$A \in \mathbb{R}^{n\times n}$ is the constant system matrix, which will be selected below,

$u\left(t\right) \in \mathbb{R}^m$ is the vector of admissible control inputs,

$B \in \mathbb{R}^{n\times m}$ is the constant matrix of control gains,

$\xi(t,x) : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is the *uncertain vector function,* assumed to be bounded as

$$\boxed{\xi^{\intercal}(t,x)Q_{\xi}\xi(t,x) \leq c_0 + x^{\intercal}Q_x x} \tag{10.3}$$

and below referred to as a **quasi-Lipschitz function.**

The number $c_0 > 0$ and positive definite quadratic matrix Q_x is supposed to be known.

We assume in this lecture that the control function u has a form of linear state feedback

$$\boxed{u\left(t\right) = Kx\left(t\right)} \tag{10.4}$$

with matrix $K \in \mathbb{R}^{m\times n}$ referred below as to a *gain matrix* which should be designed to obtain a desired behavior of the closed-loop system.

10.1.2 Problem formulation

Problem 10.1 *The problem now is to present a stabilizing control design schemes, namely, to find a gain matrix K that allow to guarantee the boundedness of all possible trajectories $\{x\left(t\right)\}_{t\geq 0}$ of the closed-loop system (10.1)-(10.4) and to estimate, adjust and minimize the "attractive ellipsoid" containing these bounded trajectories asymptotically.*

10.2 Storage function method

Consider the quadratic function

$$\boxed{V(x) = x^{\mathsf{T}} P x, \quad P = P^{\mathsf{T}} > 0} \tag{10.5}$$

referred below to as the *storage* (or energetic) function and find its total derivative along the trajectories of the closed-loop system

$$\dot{x}(t) = (A + BK)\, x(t) + \xi(t, x). \tag{10.6}$$

Using the identity

$$2x^{\mathsf{T}} S x = x^{\mathsf{T}} \left(S + S^{\mathsf{T}} \right) x,$$

we get

$$\dot{V}(x) = 2x^{\mathsf{T}} P \dot{x} = 2x^{\mathsf{T}} P \left[(A + BK)\, x(t) + \xi(t, x) \right] =$$

$$\begin{pmatrix} x \\ \xi \end{pmatrix}^{\mathsf{T}} \begin{bmatrix} P(A + BK) + (A + BK)^T & P \\ P & 0 \end{bmatrix} \begin{pmatrix} x \\ \xi \end{pmatrix}.$$

Adding and subtracting the terms $\alpha V(x)$ and $\varepsilon \xi^{\mathsf{T}}(t, x) Q_\xi \xi(t, x)$ in the right-hand side of the last equation, we obtain:

$$\dot{V}(x) = \begin{pmatrix} x \\ \xi \end{pmatrix}^{\mathsf{T}} \begin{bmatrix} P(A + BK) + (A + BK)^T P + \alpha P & P \\ P & -\varepsilon Q_\xi \end{bmatrix} \begin{pmatrix} x \\ \xi \end{pmatrix}$$

$$-\alpha V(x) + \varepsilon \xi^{\mathsf{T}}(t, x) Q_\xi \xi(t, x).$$

The application of the upper estimate (10.3) to the last term implies

$$\dot{V}(x) \leq \begin{pmatrix} x \\ \xi \end{pmatrix}^{\mathsf{T}} \begin{bmatrix} P(A + BK) + (A + BK)^T P + \alpha P & P \\ P & -\varepsilon Q_\xi \end{bmatrix} \begin{pmatrix} x \\ \xi \end{pmatrix}$$

$$-\alpha V(x) + \varepsilon \left(c_0 + x^T Q_x x \right) =$$

$$\begin{pmatrix} x \\ \xi \end{pmatrix}^{\mathsf{T}} \begin{bmatrix} P(A + BK) + (A + BK)^T P + \alpha P + \varepsilon Q_x & P \\ P & -\varepsilon Q_\xi \end{bmatrix} \begin{pmatrix} x \\ \xi \end{pmatrix}$$

$$-\alpha V(x) + \varepsilon c_0.$$

Shortly the last differential inequality can be represented as

$$\dot{V}(x) \leq \begin{pmatrix} x \\ \xi \end{pmatrix}^{\mathsf{T}} W_{\alpha,\varepsilon}\left(P, K, A \right) \begin{pmatrix} x \\ \xi \end{pmatrix} - \alpha V(x) + \varepsilon c_0, \tag{10.7}$$

where the matrix $W_{\alpha,\varepsilon}(P,K,A)$ is as follows:

$$\left.\begin{array}{c} W_{\alpha,\varepsilon}(P,K,A) := \\ \begin{bmatrix} P(A+BK)+(A+BK)^T P+\alpha P+\varepsilon Q_x & P \\ P & -\varepsilon Q_\xi \end{bmatrix}. \end{array}\right\} \quad (10.8)$$

10.3 Attractive ellipsoid

10.3.1 Definition of an attractive ellipsoid

Definition 10.1 *The ellipsoid*

$$\mathcal{E}_{\mathring{x}}(P_{attr}) := \{x \in \mathbb{R}^n : (x-\mathring{x})^{\intercal} P_{attr}(x-\mathring{x}) \leq 1\} \quad (10.9)$$

*with the center in the point $\mathring{x}$ and the ellipsoidal matrix $P_{attr} = P_{attr}^{\intercal} > 0$ is said to be **attractive** for the system (10.6) with uncertainties (10.2) and the control (10.4) if for any trajectories $\{x(t)\}_{t\geq 0}$*

$$\limsup_{t\to\infty} \, (x(t)-\mathring{x})^{\intercal} P_{attr}(x(t)-\mathring{x}) \leq 1. \quad (10.10)$$

Notice that if the attractive ellipsoid $\mathcal{E}_{\mathring{x}}(P)$ is located in the origine than $\mathring{x}=0$, then (10.9) becomes

$$\boxed{\limsup_{t\to\infty} x(t)^{\intercal} P_{attr} x(t) \leq 1} \quad (10.11)$$

and

$$\boxed{\mathcal{E}_{\mathring{x}}(P_{attr}) = \mathcal{E}_0(P_{attr}).}$$

10.3.2 Attractive ellipsoid for proportional linear feedback

Theorem 10.1 *If for the gain feed back matrix K there exist a positive definite matrix P, a matrix $A \in \mathbb{R}^{n\times n}$ and positive scalars α and ε such that*

$$\boxed{W_{\alpha,\varepsilon}(P,K,A) < 0,} \quad (10.12)$$

then we can guarantee the convergence (10.11) of all possible trajectories $\{x(t)\}_{t\geq 0}$ to the attractive ellipsoid $\mathcal{E}_0(P_{attr})$ with the ellipsoidal matrix

$$\boxed{P_{attr} = \frac{\alpha}{\varepsilon c_0} P.} \quad (10.13)$$

Proof. If the matrix inequality (10.12) holds, then from (10.7) we get

$$\dot{V}(x(t)) \leq -\alpha V(x(t)) + \varepsilon c_0,$$

implying

$$V(x(t)) \leq \frac{\varepsilon c_0}{\alpha} + \left(V(x(0)) - \frac{\varepsilon c_0}{\alpha} \right) e^{-\alpha t} \underset{t\to\infty}{\to} \frac{\varepsilon c_0}{\alpha},$$

and hence,

$$\limsup_{t\to\infty} x(t)^{\intercal} P x(t) \leq \frac{\varepsilon c_0}{\alpha},$$

or equivalently,

$$\limsup_{t\to\infty} x(t)^{\intercal} \left(\frac{\alpha}{\varepsilon c_0} P \right) x(t) \leq 1.$$

which proves (10.13). ■

Remark 10.1 *This control design scheme is rather classical and well-known for stabilization of disturbed linear control systems,i.e.,*

$$f(x,t) = Ax, \ \varphi(x^*(t), t) = 0, Q_x = 0 \text{ and } Q_u = 0$$

*(see [13], [14] and [17]). For **quasi-Lipschitz system,** satisfying (10.3), this fact is not obvious but expectable.*

10.4 Minimization of the attractive ellipsoid

To find the *"optimal"* linear feedback minimizing the attractive ellipsoid of the closed loop system (10.1)-(10.4) we will consider the following optimization problem corresponding to the minimization of the "size" of the ellipsoid $\mathcal{E}_0(P_{attr})$. When we speak about the "size" of an ellipsoid with a matrix P we do not mean its volume. A volume of an ellipsoid (or, equivalently, its determinant) in fact is a bad function for the characterization of its "size" by two following reasons:

1) since

$$\det P^{-1} = \prod_{i=1}^{N} \lambda_i(P^{-1}) \text{ and } \rho_i^2(P) = \lambda_i(P^{-1}),$$

where $\lambda_i(P^{-1})$ $(i = 1, ..., N)$ are the eigenvalues of the inverse ellipsoid matrix P^{-1} and $\rho_i(P)$ are the longitude of i-th semi-axises of the ellipsoid $\mathcal{E}_0(P)$. In view of this, we may conclude that minimization of $\det(P)$ is equivalent to minimization of its volume:

$$\mathrm{vol}(P) = \det P^{-1} = \prod_{i=1}^{n} \rho_i^2(P).$$

But, the product $\prod\limits_{i=1}^{N} \rho_i^2(P)$ admits to have a very large value of one of semi-axises, for example, $\rho_{i_0}(P)$ and all others may be very-very small! This exactly means that the designed controller guarantees a very good quality of control practically in all directions except one where it works with a very bad quality. That's why the criterion $\mathrm{tr}(P^{-1})$ is preferable since

$$\mathrm{tr}P^{-1} = \sum_{i=1}^{N} \lambda_i(P^{-1}) \leq \max_{i=1,\ldots,N} \lambda_i(P^{-1}) = \lambda_{\max}(P^{-1}),$$

and the minimization of $\mathrm{tr}P^{-1}$ guarantees, at least, the minimization of its maximum eigenvalue, and hence, this guarantees the minimization of the corresponding maximal semi-axis $\rho_{\max}(P) = \sqrt{\lambda_{\max}(P^{-1})}$ of the given ellipsoid $\mathcal{E}_0(P)$. So, the trace of the matrix P^{-1} defines the sum of squares of the ellipsoid's semi-axes.

2) The second reason, important from the numerical-computation point of view, is that $\mathrm{tr}P^{-1}$ is a linear function of the matrix P^{-1} and $\det(P^{-1})$ is not!

So, based on these comments, let us associate the optimal parameter K (10.4) of the linear feedback with the solution of the following matrix optimization problem

$$\boxed{\begin{array}{c} \mathrm{tr}(P^{-1}) \to \inf\limits_{P>0,A,K,\alpha>0,\varepsilon>0} \\ \text{subject to the matrix constraint (10.12)} \\ W_{\alpha,\varepsilon}\left(P,K,A\right) < 0. \end{array}} \tag{10.14}$$

10.5 Conversion of NMI in to LMI constraints

Notice first that matrix constraint (10.12) is bilinear (even scalar parameters α and ε are fixed) with respect to optimizing variable P, A, K. Indeed, it contains the pairs PA and PBK of matrix to be found. Recal the following fact from theory of quadratic form: the quadratic form $x^\intercal Qx$ is negative definite, that is, $x^\intercal Qx < 0$ for all $x \neq 0$ if and only if $x^\intercal (T^\intercal QT)\, x < 0$ $(x \neq 0)$ for any nonsingular matrix T $(\det T \neq 0)$. In matrix format this property looks as follows:

$$\boxed{Q < 0 \text{ equivalent } T^\intercal QT < 0 \text{ for any nonsingular matrix } T.}$$

Let us apply this fact to the matrix $W_{\alpha,\varepsilon}(P, K, A)$ selecting T as

$$T = \begin{bmatrix} P^{-1} & 0_{n\times n} \\ 0_{n\times n} & I_{n\times n} \end{bmatrix}, \ P^{-1} = \left(P^{-1}\right)^\intercal. \tag{10.15}$$

Then, in view of (10.12), we obtain

$$T^\intercal W_{\alpha,\varepsilon}(P, K, A)\, T = \begin{bmatrix} \begin{matrix} (A+BK)P^{-1} + \left[(A+BK)P^{-1}\right]^\intercal + \\ \alpha P^{-1} + \varepsilon P^{-1} Q_x P^{-1} \end{matrix} & I_{n\times n} \\ I_{n\times n} & -\varepsilon Q_\xi \end{bmatrix} < 0. \tag{10.16}$$

Introduce new matrix variables

$$X := P^{-1}, \ Y := KP^{-1}, \ Z := AP^{-1} \tag{10.17}$$

and represent (10.16) in new variables:

$$\bar{W}_{\alpha,\varepsilon}(X, Y, Z) := \begin{bmatrix} Z + BY + Z^\intercal + Y^\intercal B^\intercal + \alpha X + \varepsilon X^\intercal Q_x X & I_{n\times n} \\ I_{n\times n} & -\varepsilon Q_\xi \end{bmatrix} < 0. \tag{10.18}$$

The quadratic term XQ_xX in (10.18) may be estimated as

$$X^\intercal Q_x X < Q \Leftrightarrow Q - XQ_xX > 0,$$

which by the Schur's complement (1.6) may be equivalently represented as

$$\begin{bmatrix} Q & X \\ X^\intercal & Q_x^{-1} \end{bmatrix} > 0 \tag{10.19}$$

and, finally, the constraint (10.18), containing quadratic term, may be equivalently represented as two LMI's:

$$\boxed{\left.\begin{array}{c} \begin{bmatrix} Z + BY + Z^\intercal + Y^\intercal B^\intercal + \alpha X + \varepsilon Q & I_{n\times n} \\ I_{n\times n} & -\varepsilon Q_\xi \end{bmatrix} < 0, \\[2ex] \begin{bmatrix} -Q & -X \\ -X^\intercal & -Q_x^{-1} \end{bmatrix} < 0. \end{array}\right\}} \tag{10.20}$$

So, we are ready to formulate the following result.

Lemma 10.1 *The nonlinear (bilinear) matrix optimization problem (10.14) can be equivalently represented as a matrix optimization problem) with LMI's constraints (under the fixed positive scalar parameters α, ε, that is,*

$$\boxed{\begin{array}{c} \operatorname{tr}(X) \to \inf_{X>0,Y,Z,Q>0,\alpha>0,\varepsilon>0} \\ \textit{subject to the matrix constraints (10.20).} \end{array}} \tag{10.21}$$

If the tuple $(X^* > 0, Y^*, Z^*, \alpha^* > 0, \varepsilon^* > 0)$ is the solution of (10.21), then the optimal gain matrix K^* of the corresponding "*optimal*" linear feedback can be calculated as

$$\boxed{K^* = Y^* \left(X^*\right)^{-1},} \tag{10.22}$$

and the optimal ellipsoidal matrix P^*_{attr} is calculated as

$$\boxed{P^*_{attr} = \frac{\alpha^*}{\varepsilon^* c_0} \left(X^*\right)^{-1}.} \tag{10.23}$$

The "*best" (providing the minimal ellipsoid size) approximator* $Ax - f(x,t)$ *of nonlinearity* is

$$A^* = Z^* \left(X^*\right)^{-1}. \tag{10.24}$$

10.6 Numerical procedure for calculation of optimal feedback gain matrix

Algorithm:

1) At each step k $(k = 1, 2, ...)$ of iterations for any fixed positive scalars the constraints $\alpha_k > 0, \varepsilon_k > 0$ the matrix inequalities (10.20) becomes LMI's and the corresponding optimization problem can be effectively solved using appropriate mathematical software such as MATLAB with any SDP solver like SEDUMI or YALMIP. Let us denote by

   $$g(\alpha_k, \varepsilon_k) := \min_{X>0,Y,Z,Q>0} \operatorname{tr}(X)$$

   the corresponding minimal value.

2) The optimization of the function $g(\alpha_k, \varepsilon_k)$ with respect to parameter α_k, ε_k can be realized locally basing on some derivative-free method, for example, using the MATLAB function fminsearch. In particular,

$$\alpha_{k+1} = \alpha_k + \Delta\alpha_k, \ \Delta\alpha_k > 0,$$
$$\varepsilon_{k+1} := \varepsilon_k - \Delta\varepsilon_k, \ \Delta\varepsilon_k > 0.$$

If ε_{k+1} becomes to be negative, we return back to the previous positive value. The same should be done if the matrix optimization problem says that admissible solutions do not exist.

3) Then iterations repeat.

10.7 Practical stabilization

a) The *practical stabilization problem* [15] also requires the consideration of an optimization procedure slightly differed from the previous one. This problem consists in designing a proportional linear feedback which guarantees a convergence of all trajectories of the closed-loop system (10.1)-(10.4) into the predetermined ellipsoid $\mathcal{E}_0(P_{ref})$, where $P_{ref} > 0$ is a given positive definite matrix. Such requirement can be easily taken into account by incorporating the linear matrix inequality

$$\boxed{P^{-1} = X \leq X_{ref} = P_{ref}^{-1}} \tag{10.25}$$

into the system of inequalities (10.20).

b) Practical implementations always restricts the maximum admissible control magnitude, for example, by the following inequality

$$\boxed{\|u\|^2 < \mu,} \tag{10.26}$$

where μ is a given positive number. Taking into account the structure (10.4), this addittional constraint may be represented as

$$x^T K^T K x < \mu \quad \text{for all } x \in \mathbb{R}^n,$$

or equivalently, in the matrix format as

$$K^T K < \mu I_{n \times n} .$$

In new matrix variables (10.17) $K = YX^{-1}$ and the last matrix inequality becomes

$$\left(X^{-1}\right)^{\intercal} Y^{\intercal} Y X^{-1} < \mu I_{n\times n}\,,$$

which may be equivalently represented as

$$0 < \mu X^{\intercal} X - Y^{\intercal} Y.$$

By the Schur's complement (1.6) it takes place if and only if

$$\begin{bmatrix} \mu X^{\intercal} X & Y \\ Y^{\intercal} & I_{n\times n} \end{bmatrix} > 0,$$

which, in turn be valid if

$$\begin{bmatrix} \mu q I_{n\times n} & Y \\ Y^{\intercal} & I_{n\times n} \end{bmatrix} > 0$$

for a scalar $q > 0$, satisfying

$$X^{\intercal} X = X^2 > q I_{n\times n}.$$

This leads to the following last additional matrix inequality

$$X > \sqrt{q} I_{n\times n}\,.$$

Lemma 10.2 (on practical stabilization) *"Optimal" feedback*

$$K^* = Y^* \left(X^*\right)^{-1}$$

under specific practical conditions (10.25) and (10.26)

$$P^{-1} \leq P_{ref}^{-1} \text{ and } \|u\|^2 < \mu$$

corresponds to the solution of the following matrix optimization problem

$$\boxed{\begin{array}{c} \operatorname{tr}(X) \to \inf\limits_{X>0,Y,Z,,Q>0,\alpha>0,\varepsilon>0,q>0} \\ \text{subject to the matrix constraints (10.20)} \\ \text{and the additional ones} \\ \begin{bmatrix} \mu q I_{n\times n} & Y \\ Y^{\intercal} & I_{n\times n} \end{bmatrix} > 0,\ X > \sqrt{q} I_{n\times n}\,. \end{array}} \qquad (10.27)$$

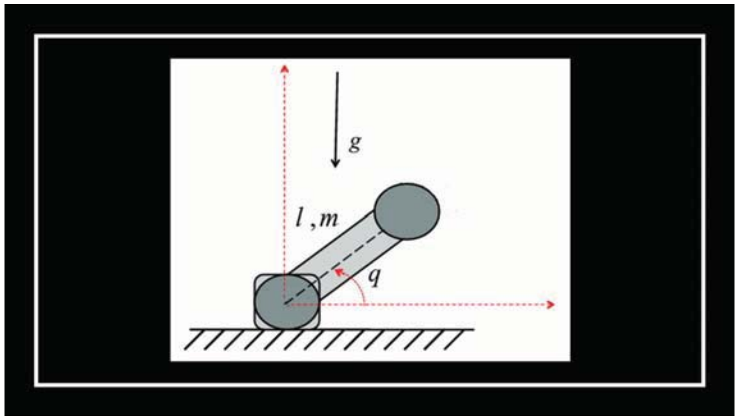

Figure 10.1: Inverted pendulum.

10.8 Illustrative Example: an inverted pendulum

In this section we consider the inverted pendulum shown in Fig.10.1.

The control to be designed is intended to maintain (stabilize) the pendulum in the vertical position. The mathematical model of the disturbed inverted pendulum can be presented as

$$ml^2\ddot{q} + mgl\sin(q) = bu + p(t), \tag{10.28}$$

where

- the position coordinates (angle) $q \in \mathbb{R}$ with associated velocities $\dot{q}$ and accelerations $\ddot{q}$ are controlled by the driving force $u \in \mathbb{R}$ with the gain $b \in \mathbb{R}$,
- m is the mass of pendulum,
- l is the distance from the pivot point to the center of mass,
- the function $p : \mathbb{R} \to \mathbb{R}$ describes the bounded exogenous disturbances

$$|p(t)| < p_0 \quad \forall t > 0.$$

The parameters of the system (10.28) are assumed be calculated with some errors

$$m = m_0(1 + \delta m), \quad l = l_0(1 + \delta l),$$

where m_0, l_0, b are given values, $\delta m, \delta l$ are small modeling errors. In the format (10.1) the original system (10.28) can be represented as as follows:

$$\dot{x} = A_0 x + Bu + \xi(t, x, u),$$

$$x = (x_1, x_2)^{\intercal} \in \mathbb{R}^2,$$

$$x_1 = \frac{\pi}{2} - q, \; x_2 = \dot{x}_1 = -\dot{q},$$

$$A = A_0 + \Delta A,$$

where

$$A_0 = \begin{pmatrix} 0 & 1 \\ m_0 g l_0 & 0 \end{pmatrix}, \; B = \begin{pmatrix} 0 \\ -b \end{pmatrix}, \; \xi(t, x) = \begin{pmatrix} 0 \\ \zeta(t, x) \end{pmatrix},$$
$$\zeta(t, x) = -\frac{p(t)}{(1+\delta m)(1+\delta l)^2} + m_0 g l_0 \left[\frac{1}{1+\delta l} \sin(x_1) - x_1 \right].$$

Notice that $\zeta(t, x)$ satisfies the quasi-Lipschitz condition

$$\zeta(t, x) \le \zeta_0 + x^T V_x^T \tilde{Q}_x V_x x$$

with

$$\zeta_0 = \frac{3p_0^2}{(1+\delta m)^2(1+\delta l)^4}, \; \tilde{Q}_x = 3m_0^2 g^2 l_0^2 \left(1 + \frac{1}{4(1+\delta l)}\right)^2, \; V_x = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

Consider the following parameters of the model

$$m_0 = 0.075 \text{ kg}, \quad \delta m = -0.02, \quad l_0 = 0.3 \text{ m}, \quad \delta l = -0.01,$$
$$b = 1, \quad g = 9.81 \text{ m/sec}^2, \quad p_0 = 0.02.$$

and restrict the maximum value of the control input inside the attractive ellipsoid by $\mu = 0.3$ (10.26). Applying the technique, presented above, we obtain

$$X^* = \begin{bmatrix} 0.0119 & 0.0128 \\ -0.0128 & -0.0199 \end{bmatrix}, \; K^* = \begin{bmatrix} -4.2606 & -3.8566 \end{bmatrix}.$$

The best (minimizing the size of attractive ellipsoid) linear approximation ΔA has been found as

$$(\Delta A)^* = Z^* (X^*)^{-1} - A_0.$$

The dynamics were simulated using the Runge-Kutta method (ODE45) within the MatLab Simulink 7.8 with $x(0) = \begin{bmatrix} \frac{\pi}{2} & 0 \end{bmatrix}^{\intercal}$. Fig.10.2 shows the evolution of the system state (angle and angular velocity) with the designed control law. Fig.10.3 depicts the obtained attractive ellipsoid.

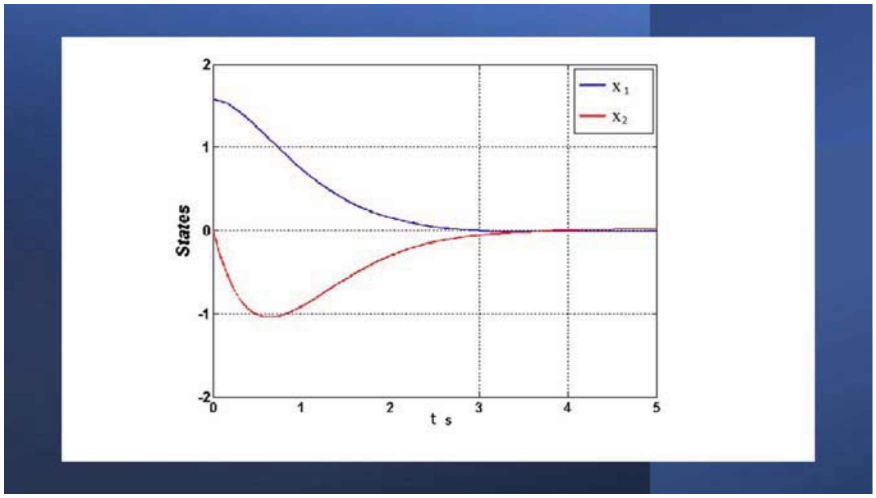

Figure 10.2: Pendulum trajectories.

10.9 Exercises

Exercise 10.1 *For the dynamic controlled system*

$$\frac{d}{dt}\begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} = \begin{pmatrix} \bar{x}_1 + \operatorname{arctg}(\bar{x}_2) \\ -\bar{x}_1 - \sin \bar{x}_2 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix} \bar{u}(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \bar{\xi}_x(t),$$

$$\bar{\xi}_x(t) = 0.1 \sin(10t),$$

design the control feedback using AEM concept in the tracking problem for the desired trajectory $x^*(t) \in \mathbb{R}^2$ satisfying

$$\ddot{x}^*(t) + \omega^2 x^*(t) = 0, \ \omega = 2,$$

$$x^*(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \ \dot{x}^*(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

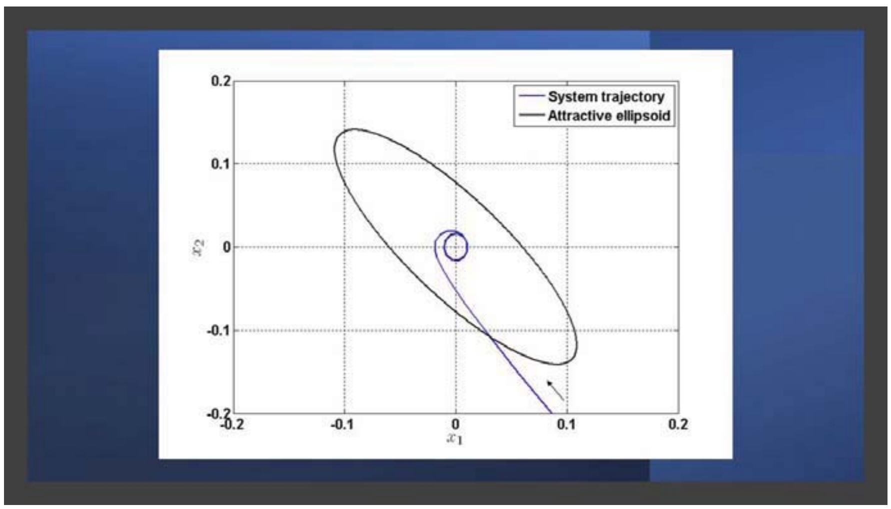

Figure 10.3: Attractive ellipsoid.