Adaptive Friction Compensation Using Neural Network Approximations

S. N. Huang, K. K. Tan, and T. H. Lee

Abstract—We present a new compensation technique for a friction model, which captures problematic friction effects such as Stribeck effects, hysteresis, stick-slip limit cycling, pre-sliding displacement and rising static friction. The proposed control utilizes a PD control structure and an adaptive estimate of the friction force. Specifically, a radial basis function (RBF) is used to compensate the effects of the unknown nonlinearly occurring Stribeck parameter in the friction model. The main analytical result is a stability theorem for the proposed compensator which can achieve regional stability of the closed-loop system. Furthermore, we show that the transient performance of the resulting adaptive system is analytically quantified. To support the theoretical concepts, we present dynamic simulations for the proposed control scheme.

Index Terms—Adaptive systems, compensation, friction, neural networks.

I. INTRODUCTION

FRICION is a natural resistance to relative motion between two contacting bodies. When a controller is designed without consideration of the friction, the closed-loop systems show steady-state tracking errors and/or oscillations. For eliminating or reducing the effects of the friction force, model-based friction compensation technique is an active control strategy. Much effort has been done on various control laws for friction compensations. Leonard and Krishnaprasad [1] proposed an adaptive friction compensation to improve performance for tracking errors; however, the Stribeck effect was not modeled. In Friedland and Park [2], the Coulomb friction effect was discussed, with theoretical and simulation results. In Li and Cheng [3], nonlinear adaptive friction compensation was shown to improve tracking performance compared to a PID controller by modeling friction as a static nonlinear dynamics. For the same static friction model, Tan et al. [4] presented a sliding mode controller for friction compensation. In Dupont [5], a PD controller was designed for avoiding the friction with known parameters. In Canudas de Wit et al. [6], a new dynamical friction model was presented and a nonlinear friction compensator with fixed parameter estimates was designed. Moreover, Canudas de Wit and Lischinsky [7] showed that the stability design using the strictly positive-real (SPR) condition is conservative. The work in Canudas de Wit and Lischinsky [7] also includes adaptive design in which only lumped parameter is considered. In Otten et al. [8], a neural network based controller was used for adaptive friction compensator where the authors considered a static friction model. In Vedagrabba et al. [9], adaptive control was investigated with providing estimates of the mass and two dynamic parameters. In Hirschorn and Miller [10], a continuous dynamic controller was proposed for a more general class of nonlinear systems with the friction model in [6]. However, the parameters of modeling the Stribeck effects are assumed to be known. Based on the friction model of [6], Panteley et al. [11] presented an adaptive control scheme plus sliding mode with more of the friction parameters to be considered than those in previous results. Unfortunately, the Stribeck effects are considered as a disturbance term.

Since neural networks have a capability of approximating nonlinear functions, it is natural to use them in friction estimates. Much work has been done in this area (see [8], [12]–[14], [19]–[22]). However, theoretical analysis is scarce if any, in their results [8], [12]–[14], [21], [22]. In cases where the system properties ([19], [20]) were discussed the friction models used are often simple (static friction model) and may not represent the actual system accurately. Motivated by the fact that better results can be obtained when more properties of the studied system are exploited, in this paper our results are based on the friction model in [6] which is the most popular model used in the literature. The proposed control utilizes a PD control structure and an estimate of the friction force. Since the control requires an estimate of the dynamic friction, we design an adaptive scheme for more of the friction parameters. In this adaptive scheme, the Radial Basis Functions (RBFs) are used to compensate the effects of the unknown nonlinearly-occurring Stribeck parameter in the friction model. The main analytical result is a stability theorem for the proposed compensator which can achieve regional stability of the closed-loop system. Furthermore, we show that the transient performance of the resulting adaptive system is analytically quantified. Simulations show converging estimates for unknown parameters within the friction model and the improved tracking performance.

II. PROBLEM STATEMENT

The mechanical system under investigation is a mass moving in one dimension with friction present between the mass and the supporting surface. The equation for this model is described as follows:

\[ m \ddot{x} = u - F \]  

where

- \( m \) is the mass;
- \( x \) is the mass position;
- \( \dot{x} \) is the mass velocity;
- \( u \) is the control;
- \( F \) is the friction force.

The goal is to choose a control law to accurately track a desired trajectory. Since we intend to design the control scheme to compensate the effects of friction, this requires a suitable friction model. A physics-based model, proposed by [6], is complex enough to capture dynamic friction effects, such as Stribeck effect, hysteresis, stick-slip limit cycling, pre-sliding displacement and rising static friction. In this paper, we use this model. The contact surface effects are lumped into an average asperity deflection \( z \) that is modeled by

\[ \ddot{z} = \frac{1}{h(\dot{x})} \dot{x} - \frac{1}{h(\dot{x})} z. \]

The first term gives a deflection that is proportional to the integral of the relative velocity. The second term asserts that the deflection \( z \) approaches a steady-state value \( z_s \), given by

\[ z_s = h(\dot{x}) \text{sign}(\dot{x}) \]
Define now a control input as

$$u = K_v \dot{x} + \dot{m}(\Lambda \dot{e} + \ddot{x}_d) + \dot{\hat{F}}$$  \hspace{1cm} (10)$$

where $K_v > 0$ is constant $\hat{m}$ and $\hat{F}$ are estimates of $m$ and $F$, respectively. Obviously, if $\hat{m} = m$, $\hat{F} = F$, then the control (10) would lead to the closed-loop system expressed as $m\ddot{e} = -K_v \dot{x}$, i.e., the known friction could be well compensated and the resulting system is asymptotically stable. Unfortunately, the friction $F$ is unknown a priori in practice. In addition, it is difficult to obtain the precise value of $m$. However, the controller (10) suggests indeed that a well estimated function $\hat{F}$ of $F$ could be used to improve the tracking performance. Motivated by this, it is a reasonable approach to utilize neural networks to model the unknown friction force. For that purpose, RBF networks are employed. The main property of a RBF used here for estimation purposes is the function approximation property (see [15] and [17]–[19]). Much work has been done on the neural controllers focusing on the type of $\dot{x} = f(x) + g(x)u$ with assuming that $f(x), g(x)$ to be smooth functions (see [16], [23], [25]). From the model (6), $F$ including the terms $\text{sign}(\dot{x}), [\dot{x}]$ and the perturbation $\epsilon$ (nonmeasurable), are not smooth functions. Thus, the previous results are not applicable.

Let $f(\chi)$ be a smooth function from $R^n$ to $R^p$. Then, given a compact $S \subset R^n$ and a positive number $\varepsilon_M$, there exists a RBF system such that

$$f(\chi) = \sum_{i=0}^m \omega_i \phi_i(\chi) + \varepsilon$$  \hspace{1cm} (11)$$

where $\omega_i$ is the representative value vector and $\phi_i(\chi)$ is the radial basis function with $||\chi|| < \varepsilon_M$ for all $\chi \in S$. Since $F_v + (F_v - F_v)e^{-\sigma_1 ||\chi||^2}$, denoted as $f(\chi)$, is a nonlinear smooth function (unknown), on a compact set $U_v = \{X \mid ||X_d - X|| < M_\varepsilon\} \pi X = [x, \dot{x}]^T, X_d = [x_\text{d}, \dot{x}_\text{d}]^T$ it may be represented by RBF with constant “ideal” weights $\omega_{j,i}, i = 0, 1, 2, \ldots, m$ and a sufficient number of basis functions $\phi_{j,i}(\cdot)$, i.e.

$$f(\chi) = \sum_{i=0}^m \omega_{j,i} \phi_{j,i}(\chi) + \varepsilon_j$$  \hspace{1cm} (12)$$

where $\varepsilon_j$ is the RBF approximation error satisfying $||\varepsilon_j|| \leq \varepsilon_{JM}$ with constant $\varepsilon_{JM}$, and $\phi_{j,i} (\cdot)$ is given by

$$\phi_{j,0} = 1, \phi_{j,i} (\chi) = \exp \left( -\frac{||\chi - c_i||^2}{\sigma_i^2} \right)$$  \hspace{1cm} (13)$$

where $c_i$ represents the center of the $i$th basis function, and $\sigma_i$ represents the spread of the basis function. For the sake of more tractable analysis, the parameters $c_i, \sigma_i$ are chosen a priori and kept fixed. We shall indicate in the simulation section how to choose a basis set specifically for the friction compensation. Notice that the set $U_v$ and the bounding constants $\varepsilon_{JM}$ can be arbitrary large. Substituting (12) into (9) we have

$$m\ddot{r} = m(\Lambda \dot{e} + \dddot{x}_d) + \sigma_2 \dot{x} + \left( \sum_{i=0}^m \omega_{j,i} \phi_{j,i} + \varepsilon_j \right) \text{sign}(\chi)$$

$$- u + F_\text{d}(\chi, e) = \theta^T Y(\dot{r}, \dddot{x}_d, \chi) + \left( \sum_{i=0}^m \omega_{j,i} \phi_{j,i} + \varepsilon_j \right) \text{sign}(\chi)$$  \hspace{1cm} (14)$$

where $\theta^T = [m, \sigma_2], Y^T (\dot{r}, \dddot{x}_d, \dot{x}) = [\Lambda \dot{e} + \dddot{x}_d, \dot{x}]$. 

Fig. 1. Basic friction of a mechanical system.

while $\dot{x}$ is constant. The function $h$ is given by

$$h(\dot{x}) = \frac{F_v + (F_v - F_v)e^{-\sigma_1 ||\dot{x}||^2}}{\sigma_0}$$  \hspace{1cm} (4)$$

where

- $F_v$ Coulomb friction level;
- $\sigma_0$ level of the stiction force;
- $\sigma_1$ stiffness;
- $\sigma_2$ viscous friction coefficient.

Lemma 2.1 [6]: The internal friction state $z$ is bounded. The friction force can also be written in terms of $e = \dot{z} - z$ as

$$F = \sigma_0 \dot{z} + \sigma_1 \dot{z} + \sigma_2 \dot{z}$$  \hspace{1cm} (5)$$

where

- $\sigma_0$ stiffness;
- $\sigma_1$ damping coefficient;
- $\sigma_2$ viscous friction coefficient.

The first part $\sigma_0 \dot{z} + [F_v + (F_v - F_v)e^{-\sigma_1 ||\dot{x}||^2}] \text{sign}(\dot{x})$ is a static function of the velocity. The second part $\sigma_1 e^{-\sigma_1 ||\dot{x}||^2}$ is scaled by the error $e$ due to the dynamic perturbations in friction. It can also be bounded as

$$|F_d(\dot{x}, e)| = \left| \sigma_0 e^{-\sigma_1 ||\dot{x}||^2} \right| \leq \Delta_1 ||\dot{x}|| + \Delta_2$$  \hspace{1cm} (7)$$

where $e$ is bounded since $\dot{z}$ and $z$ are bounded.

Given a desired trajectory $x_d$ the tracking error is $e = x_d - x$. In standard use in robotics is the filtered tracking error

$$r = \Lambda \dot{e} + \dot{e}$$  \hspace{1cm} (8)$$

where $\Lambda > 0$ is a design parameter. Differentiating $r(t)$, (8) may be written in terms of the filtered tracking errors as

$$m\dddot{r} = m(\Lambda \dot{e} + \dddot{x}_d) + \sigma_2 \dot{x} + \left[ F_v + (F_v - F_v)e^{-\sigma_1 ||\dot{x}||^2} \right] \text{sign}(\dot{x})$$

$$- u + F_\text{d}(\dot{x}, e).$$  \hspace{1cm} (9)$$
III. MAIN RESULTS

In this section, we develop an adaptive nonlinear compensator which consists of a PD control and an adaptive friction estimator. The design objective is to make the effects of the error due to friction estimation small. The controller is of the form

\[
u = K_v \hat{r} + \theta^T Y(\hat{\dot{x}}, \hat{x}, \dot{x}) + \sum_{i=0}^{p} \hat{\omega}_i \dot{\phi}_i \sign(\hat{\dot{x}}) + \Delta_1 \left| \hat{\dot{x}} \right| \sign(r)
\]

where
- \(\hat{\theta}\) estimate of \(\theta\);
- \(\hat{\omega}_i\) estimates of the ideal weights \(\omega_i\);
- \(\Delta_1\) estimates of \(\Delta_1\).

Substituting the control (15) into (14), we have

\[
\dot{\mathbf{m}} \dot{\mathbf{r}} = -K_v \dot{\mathbf{r}} + \theta^T Y(\hat{\dot{x}}, \hat{x}, \dot{x}) + \left( \sum_{i=0}^{p} \hat{\omega}_i \dot{\phi}_i + \varepsilon_f \right) \times \sign(\hat{\dot{x}}) - \Delta_1 \left| \hat{\dot{x}} \right| \sign(r) + F_d(\dot{x}, \dot{\mathbf{e}})
\]

where \(\hat{\theta} = \theta - \hat{\theta}\), \(\hat{\omega}_i = \omega_i - \hat{\omega}_i\).

**Theorem 3.1:** Assume the desired trajectories \(X_d\) be bounded. Let the control input be given by (15) and estimation algorithms provided by

\[
\begin{align*}
\dot{\hat{\theta}} &= k_0 \left[ r \hat{\theta} - k_1 \hat{\theta} \right] \\
\hat{\omega}_i &= k_0 \left[ \phi_i \sign(r) - k_1 \hat{\omega}_i \right], \quad i = 0, 1, \ldots, p \quad (17) \\
\dot{\hat{\Delta}}_1 &= k_0 \left[ \left| \hat{\dot{x}} \right| r - k_1 \hat{\Delta}_1 \right] \quad (19)
\end{align*}
\]

where \(k_0, k_1 > 0\). Then the error \(r(t), \hat{\theta}, \hat{\Delta}_1\), and RBF weights are uniformly ultimately bounded. Moreover, the uniform ultimate boundedness of the tracking error \(r(t)\) can be kept as small as desired by increasing \(k_0\) and \(K_v\).

**Proof:** The proof is similar to that presented by [24]. It includes two steps. We first assume that the system states remain in a compact set \(U_r\) and the system signals are bounded, then, we show that, by properly choosing controller parameters, the system states indeed remain in the compact set \(U_r\).

Step 1: Let

\[
V = \frac{1}{2} \left[ m \dot{r}^2 + \frac{1}{k_0} \hat{\theta}^T \hat{\theta} + \frac{1}{k_0} \hat{\Delta}_1^2 + \frac{1}{k_0} \sum_{i=0}^{p} \hat{\omega}_i^2 \right].
\]

Taking the time derivative of \(V\) and using (7), it can be shown that

\[
\dot{V} \leq -K_v r^2 + \hat{\theta}^T Y(\dot{\hat{x}}, \hat{x}, \dot{x}) \dot{r} + \sum_{i=0}^{p} \hat{\omega}_i \dot{\phi}_i \sign(\hat{\dot{x}}) r
\]

\[
- \Delta_1 \left| \dot{\hat{x}} \right| \sign(r) + \Delta_1 \left| \hat{\dot{x}} \right| r + \Delta_2 \left| r \right| + \varepsilon_f r
\]

\[
+ \frac{1}{k_0} \hat{\theta}^T \hat{\theta} + \frac{1}{k_0} \Delta_1 \hat{\Delta}_1 + \frac{1}{k_0} \sum_{i=0}^{p} \hat{\omega}_i \dot{\phi}_i
\]

\[
= -K_v r^2 + \hat{\theta}^T Y(\dot{\hat{x}}, \hat{x}, \dot{x}) + \sum_{i=0}^{p} \hat{\omega}_i \dot{\phi}_i \sign(\hat{\dot{x}}) r
\]

\[
+ \Delta_1 \left| \dot{\hat{x}} \right| r + \Delta_2 \left| r \right| + \varepsilon_f r
\]

\[
- \frac{1}{k_0} \hat{\theta}^T \hat{\theta} - \frac{1}{k_0} \Delta_1 \hat{\Delta}_1 - \frac{1}{k_0} \sum_{i=0}^{p} \hat{\omega}_i \dot{\phi}_i.
\]

Applying (17)–(19) to (21), we get

\[
\dot{V} \leq -K_v r^2 + k_1 \hat{\theta}^T \hat{\theta} + k_1 \Delta_1 \hat{\Delta}_1
\]

\[
+ k_1 \sum_{i=0}^{p} \hat{\omega}_i \dot{\phi}_i + (\Delta_2 + \varepsilon_f r) r
\]

\[
= -K_v r^2 + k_1 \hat{\theta}^T \hat{\theta} - k_1 \hat{\theta}^T \hat{\theta} + k_1 \Delta_1 \hat{\Delta}_1 - k_1 \hat{\Delta}_1^2
\]

\[
+ k_1 \sum_{i=0}^{p} (\hat{\omega}_i \dot{\phi}_i - \hat{\omega}_i \dot{\phi}_i) + (\Delta_2 + \varepsilon_f r) r.
\]

By virtue of \(2 \alpha^T \beta \leq \frac{\eta \alpha^T \alpha + (1/\eta) \beta^T \beta}{2} \) (\(\eta = \text{constant} > 0\)), we have

\[
\hat{\theta}^T \hat{\theta} \leq \frac{1}{2} \hat{\theta}^T \hat{\theta} + \frac{1}{2} \hat{\theta}^T \hat{\theta}
\]

\[
\hat{\Delta}_1 \Delta_1 \leq \frac{1}{2} \hat{\Delta}_1^2 + \frac{1}{2} \Delta_1^2
\]

\[
\hat{\omega}_i \dot{\phi}_i \leq \frac{1}{2} \hat{\omega}_i^2 + \frac{1}{2} w_i^2 \hat{\phi}_i
\]

\[
(\Delta_2 + \varepsilon_f r) r \leq \frac{1}{2\eta_0} (\Delta_2 + \varepsilon_f r)^2 + \frac{\eta_0}{2} r^2
\]

\[
\dot{V} \leq -K_v r^2 + \left( \frac{\eta_0}{2} r^2 - \frac{k_1}{k_0} \hat{\theta}^T \hat{\theta}ight)
\]

\[
- \frac{k_1}{2} \hat{\Delta}_1 - \frac{k_1}{2} \sum_{i=0}^{p} \hat{\omega}_i \dot{\phi}_i + k_1 \Delta_1 \hat{\Delta}_1
\]

\[
+ k_1 \sum_{i=0}^{p} w_i^2 \hat{\phi}_i + \frac{1}{2\eta_0} (\Delta_2 + \varepsilon_f r)^2.
\]

We choose \(\eta_0 < 2K_v\) and let

\[
\hat{\lambda}_1 = \min \left\{ \frac{K_v}{m}, \frac{2K_v}{k_0} \right\} > 0
\]

\[
\hat{\lambda}_2 = \frac{k_1}{2} \hat{\theta}^T \hat{\theta} + \frac{k_1}{2} \Delta_1^2 + \frac{k_1}{2} \sum_{i=0}^{p} w_i^2 \hat{\phi}_i
\]

\[
+ \frac{1}{2\eta_0} (\Delta_2 + \varepsilon_f r)^2 > 0.
\]

Thus, we obtain

\[
\dot{V} \leq -2 \hat{\lambda}_1 V + \hat{\lambda}_2.
\]

Hence

\[
V(t) \leq \frac{\hat{\lambda}_2}{2\hat{\lambda}_1} + \left[ V(0) - \frac{\hat{\lambda}_2}{2\hat{\lambda}_1} \right] e^{2\hat{\lambda}_1 t}, \quad \forall t \geq 0.
\]

From (28) we conclude that \(r\) and parameter estimates are uniformly ultimately bounded. Furthermore, since

\[
\frac{1}{2} m \dot{r}^2 \leq V
\]

we have

\[
r^2 \leq \frac{\hat{\lambda}_2}{m \hat{\lambda}_1} + \frac{1}{m} \left[ 2V(0) - \frac{\hat{\lambda}_2}{m \hat{\lambda}_1} \right] e^{-2\hat{\lambda}_1 t}, \quad \forall t \geq 0
\]

or \(r\) is uniformly ultimately bounded with respect to the set

\[
G = \left\{ r \in \mathbb{R} : |r| \leq \sqrt{\frac{\hat{\lambda}_2}{m \hat{\lambda}_1}} \right\}.
\]

Note from (31) that arbitrarily small error bound of \(|r(t)|\) may be achieved by selecting large \(\hat{\lambda}_1\). This implies that arbitrarily small error bound of \(|r(t)|\) may be achieved by selecting large \(k_0\) and \(K_v\). This completes the proof of Step 1.
Step 2: As it is already mentioned, the results of the paper are valid as \( X \in \mathcal{U}_0 = \{ X \mid \| X_d - X \| \leq M_0 \} \). To verify that \( X \in \mathcal{U}_0 \), \( \forall t \geq 0 \) we have to analyze set \( r \). From (30), we know

\[
|e| \leq \frac{\bar{\lambda}_2}{m \lambda_1} + \frac{1}{m} \left[ 2V(0) - \frac{\bar{\lambda}_2}{\lambda_1} \right] e^{-\frac{\bar{\lambda}_2}{m} t}
\]

\[
|e| \leq \frac{\bar{\lambda}_2}{m \lambda_1} + \sqrt{\frac{2V(0)}{m}} e^{-\frac{\bar{\lambda}_2}{m} t}, \quad \forall t \geq 0.
\]

Since (8) holds, we get

\[
|e| \leq e^{-\frac{\bar{\lambda}_2}{m} t} |e(0)| + \int_0^t e^{-\frac{\bar{\lambda}_2}{m} (t-\tau)} |e| \, d\tau.
\]

(33)

Considering (32) and \( \sqrt{(1/m) 2V(0)} \leq \sqrt{(2V(0)/m) \bar{\lambda}_2} \), we have

\[
|e| \leq e^{-\frac{\bar{\lambda}_2}{m} t} |e(0)| + \frac{\sqrt{\bar{\lambda}_2}}{m \lambda_1} + \frac{\sqrt{2V(0)}}{m} \int_0^t e^{-\frac{\bar{\lambda}_2}{m} (t-\tau)} d\tau
\]

\[
|e| \leq |e(0)| + \frac{\sqrt{\bar{\lambda}_2}}{m \lambda_1} + \frac{\sqrt{2V(0)}}{m}, \quad \forall t \geq 0.
\]

(34)

Noting that

\[
|\dot{e}| \leq |\dot{e}| + \bar{\lambda}|e| \leq \bar{\lambda}|e(0)| + 2 \left[ \frac{\bar{\lambda}_2}{m \lambda_1} + \frac{\sqrt{2V(0)}}{m} \right]
\]

(35)

we obtain

\[
\| X_d - X \| = |e| + |\dot{e}| \leq (\Lambda + 1) |e(0)|
\]

\[
+ \left( 2 + \frac{1}{\Lambda} \right) \left[ \frac{\bar{\lambda}_2}{m \lambda_1} + \frac{\sqrt{2V(0)}}{m} \right]
\]

\[
\leq (\Lambda + 1) |e(0)| + |e| + |\dot{e}|
\]

\[
+ \left( 2 + \frac{1}{\Lambda} \right) \left[ \frac{\bar{\lambda}_2}{m \lambda_1} + \frac{\sqrt{2V(0)}}{m} \right]
\]

\[= (\Lambda + 1) \| X_d(0) - X(0) \|
\]

(36)

where \( \Gamma(0)^2 = \sum\bar{\lambda}_2 + \sum\bar{\lambda}_1 + \sum\bar{\lambda}_0 \), since \( \sum\bar{\lambda}_2 \leq \sum\bar{\lambda}_1 \). Since

\[
|e| \leq \Lambda|e| + |\dot{e}| \leq \Lambda_{\max} |e| + |\dot{e}|
\]

(37)

we have

\[
|e(0)| \leq \Lambda_{\max} |e(0)| + |\dot{e}(0)|
\]

\[
= \Lambda_{\max} \| X_d(0) - X(0) \|.
\]

(38)

Substituting (38) into (36), we obtain

\[
\| X_d - X \| \leq (\Lambda + 1 + \frac{2 + (1/\Lambda) \Lambda_{\max}}{\Lambda}) \| X_d(0) - X(0) \|
\]

\[
+ \left( 2 + \frac{1}{\Lambda} \right) \left[ \frac{\bar{\lambda}_2}{m \lambda_1} + \frac{\sqrt{\| \Gamma(0) \|}}{\sqrt{m k_0}} \right]
\]

(39)

Define a compact set

\[
\mathcal{U}_{\delta_0} = \left\{ X(0) \mid \| X_d(0) - X(0) \| < \delta_0 \right\}
\]

\[
\left\{ X(0) \mid \| X_d(0) - X(0) \| < \delta_0 ; \Lambda + 1 + \frac{2 + (1/\Lambda) \Lambda_{\max}}{\Lambda} \delta_0 < \delta \right\}.
\]

(40)

Then, for the initial condition \( X(0) \in \mathcal{U}_{\delta_0} \), and bounded initial \( \theta(0), \Delta_1(0), \Delta_{i}(0), \dot{\Delta}_{i}(0), i = 0, 1, \ldots, p \), there exist \( k_0, K_0 \) such that

\[
\| X_d - X \| < \varepsilon
\]

(41)

since \( (2 + (1/\Lambda)) \sqrt{(\bar{\lambda}_2/m \lambda_1) + (\| \Gamma(0) \|/\sqrt{m k_0})} \) can be made arbitrarily small by selecting large \( k_0 \) and \( K_0 \). Therefore, for the initial condition \( X(0) \in \mathcal{U}_{\delta_0} \subset \mathcal{U}_0 \), and bounded \( \theta(0), \Delta_1(0), \Delta_{i}(0), \dot{\Delta}_{i}(0), i = 0, 1, \ldots, p, \) the controller parameter is chosen such that \( k_0 \geq k_0, K_0 \geq K_0^\tau, \) then the system state \( X = [x, \dot{x}]^T \) indeed stays in \( \mathcal{U}_0 \) for all time. This completes the proof of Step 2.

Remark 3.1: It is worth noting that Theorem 3.1 guarantees the boundedness of the closed-loop system in the sense of practical stability, i.e., the initial conditions are required to meet \( x(0) \in \mathcal{U}_{\delta_0} \), and
the states stay in the compact sets $U_x$. This is reasonable because the neural network approximation is only valid on compact sets. From (40), we can compute $U_x$ explicitly. Therefore, it is not difficult to find the conditions such that the closed-loop system is bounded.

Theorem 3.1 only ensures the boundedness of the signals in the closed-loop system, no transient performance is discussed. In practical applications, the transient performance is often more important. Now we have the following theorem to discuss this problem.

**Theorem 3.2:** For the closed-loop system (1), (15), (17)–(19), if $X(0) \in U_x$, $k_0 \geq k_0^*$ and $K_v \geq K_v^*$, then the $L_\infty$ tracking error bound is

$$
\sup_{t \geq 0} |e(t)| \leq \left( \frac{\lambda_{\max}}{\lambda} + 1 \right) \|X_d(0) - X(0)\| + \frac{1}{\lambda} \sqrt{\frac{\lambda_2}{m\lambda_1}} \left[ \frac{1}{m k_0} (\hat{\tau}(0) \hat{\theta}(0) + \hat{\Delta}_1(0)^2 + \sum_{j=0}^{p} \hat{w}_j(0)^2) \right]
$$

**Proof:** Theorem 3.1 guarantees that $X \in U_x, \forall x \geq 0$. Therefore, RBF approximation (12) is valid. Therefore, inequalities (34), (38) hold. From (34), the $L_\infty$ tracking error bound can be derived from the equation shown at the bottom of the page.

**Remark 3.2:** It is shown from (42) that large initial errors $\|X_d(0) - X(0)\|$ and $\hat{w}_j(0)$ may lead to a large tracking error during the initial period of adaption. However, tracking error may be reduced by choosing $K_v$ and $k_0$. One way to reduce initial errors $\|X_d(0) - X(0)\|$ is to set them as zeros by appropriately initializing the reference trajectories $x_d(0), \dot{x}_d(0)$ using the method of [26].

**Remark 3.3:** The control law in (15) can interpreted as an adaptive sliding mode controller since $r$ is a standard sliding surface. The term $\Delta_1 |\dot{r}| \text{sign}(r)$ is to compensate the system uncertainty, which is a similar design as in the sliding mode control.

IV. SIMULATION

In this section, we present simulation studies to illustrate the performance of the adaptive friction compensator. The following friction model parameters are used: $m = 0.59$, $\sigma_0 = 10^5$, $\sigma_1 = \sqrt{10^5}$, $\sigma_2 = 0.4$, $F_e = 1$, $F_s = 1.5$, and $v_s = 0.001$.
The desired positioning trajectory is shown in Fig. 2. To apply the proposed controller, the RBF functions are first designed. The training techniques could be used for appropriately choosing \( c_i, \sigma_i \) of the RBF basis. For the RBF basis \( \phi_j \), we train the RBF network to map the function \( e^{-\frac{(x-x_j)^2}{\sigma_j}} \), where the estimate of \( \dot{x} \) is chosen as \( \dot{x} = 0.002 \) which is 50% greater than the true parameter, and the estimates \( F_i = 0.5 \) and \( F_i - F_i = 0.25 \) which are 50% less than the true parameters. After training, the RBF basis \( \phi_j \) is obtained, that is \( \rho = 4, [c_1, c_2, c_3, c_4]^T = [0.0006, 0.0026, 0, 0.001]^T \). We also choose \( \sigma_i = 0.0012, k_0 = 0.3, \lambda = 8, \kappa_i = 1, k_i = 1 \). The initial RBF weights \( w_{ji}(0) \) are set to zero and \( \Delta_j(0) = 0.1 \). Fig. 3 shows the control results. It can be seen that the tracking error is decreased significantly after \( t = 3 \) s. As the learning process continues, the tracking error is further reduced to about 0.006 after \( t = 5 \) s. The reason for this is that the RBF compensator can estimate the unknown friction terms. The RBF approximation capability is shown in Fig. 4. This confirms our theoretical analysis. Compared with a PD controller, the gains of the PD controller are chosen the same as that of the PD part in the proposed control for fairness of comparison. The result is shown in Fig. 5 and the performance is poor.

V. CONCLUSION

This paper has presented the development of a new adaptive compensator for a friction model. The compensator is especially useful for handling the friction force which exhibit significant nonlinearities in motor controls. The stability of the controlled system has been guaranteed by Lyapunov theory. The effectiveness of the proposed control scheme has also been highlighted with a simulation experiment.

REFERENCES

Combined Heuristic Knowledge and Limited Measurement Based Fuzzy Logic Antiskid Control for Railway Applications

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Abstract—In modern railway applications, the prevention of wheel skid is very important. This is because wheel skid can lead to an increase in noise and vibration from wheels with flat points, as well as an increased braking distance. However, conventional antiskid control has problems because the train wheel adhesion and skid characteristics are difficult and time consuming to accurately model. In addition, adequate measured numerical data describing wheel skid is difficult and expensive to obtain from actual railway systems. Therefore, a fuzzy logic based antiskid controller was implemented, where both linguistic and numerical system information could be used. In this paper, the design and implementation of the fuzzy logic controller is described. Results show that the antiskid controller has a very good performance, and performs better than a conventional controller. The described controller is currently running in Mitsubishi Electric railway brake sets in both Japan and overseas.

Index Terms—Antiskid, control, fuzzy logic, heuristic, railway.

I. INTRODUCTION

In railway traction, wheel skid may occur during the braking of a train. This can lead to flattened wheels and track damage, with a resultant increase in audible noise and vibration. Hence, the prevention of wheel skid with an antiskid control system is very important in train systems, especially for modern high-speed trains.

Antiskid control is a difficult problem due to the highly nonlinear, time-varying, and complex nature of the adhesion characteristics between the track and wheel. Due to the nonlinear and unknown characteristics of the wheel adhesion model, simplifications are made in the modeling process for conventional controllers. Hence conventional antiskid controllers are found to have a relatively low performance.

Fuzzy logic based modeling offers advantages for the control of complex systems and allows complex relations to be represented without mathematical models using a linguistic approach [1]–[4]. Furthermore, a fuzzy model can be developed not only from measured system data, but also by using expert knowledge in the form of linguistic rules [1]. This distinguishes it from other approaches such as neural network modeling [5] or evolutionary computation techniques [6], which rely exclusively on numerical or measured system data to form a model, and emphasize quantitative accuracy.

Measured data describing wheel skid in an actual railway system is difficult and expensive to obtain. Therefore, only a limited amount of training data is feasible. Nevertheless, linguistic rules describing wheel skid and its control can be obtained from human experts who are familiar with such systems. Unlike conventional modeling and control, this linguistic information can be directly incorporated in the control design, together with the knowledge obtained from the available train data. This means that by using fuzzy modeling, effective use is made of all available information.

Thus, in this paper, a fuzzy logic (FL) based antiskid controller for railway systems is designed. The FL antiskid controller is found to have a relatively low performance.

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