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Passivity-Based PI Control of First-Order Systems with I/O Communication Delays: a frequency domain analysis

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In this paper, the PI control of first-order linear passive systems through a delayed communication channel is revisited in light of the relative stability concept called σ-stability. Treating the delayed communication channel as a transport PDE, the passivity of the overall control-loop is guaranteed, resulting in a closed-loop system of neutral nature. Spectral methods are then applied to the system to obtain a complete stability map. In particular, we perform the $\mathcal{D}$-subdivision method to declare the exact σ-stability regions in the space of PI parameters. This framework is then utilized to analytically determine the maximum achievable exponential decay rate σ_d^* of the system while achieving the PI tuning as explicit function of σ_d^* and system parameters.

Keywords: Time-delay systems; passivity-based control; linear systems.

1. Introduction

Passivity-based control relies on the fact that the power-preserving interconnection of two passive subsystems yields again a passive system (Ortega, Loría, Nicklasson, & Sira-Ramirez, 1998). Lyapunov stability of the interconnected system follows from passivity while asymptotic stability is usually achieved by adding appropriate damping. Due to its simplicity and robustness, passivity-based control has attracted researchers and practitioners in the control community for several decades, e.g. (Byrnes, Isidori, & Willems, 1991; Hill & Moylan, 1976; van der Schaft, 2000; Willems, 1972a, 1972b; Youla, Castriota, & Carlin, 1959).

However, if a delayed communication channel stands between the plant and the controller, as in a typical control scenario, passivity arguments fail due to non-passive properties of the channel. Here, the loss of passivity follows from the fact that the Nyquist plot of a pure delay does not lie in the right-hand side of the complex plane. In their seminal work, inspired from the study of transmission lines Anderson and Spong (1989) proposed a useful modification of the communication channel to remedy the aforementioned design problem. More precisely, the communication channel is transformed into a passive system, thus recovering the simplicity and effectiveness of the passivity-based design. Such transformation is easily implemented by applying a linear map at the channel endpoints. This idea has been discussed in many contributions and has given rise to an outstanding number of proposals addressing optimality issues, applications in the field of robotics, motor control, among other studies. The reader is referred to Nuño, Basañes, and Ortega (2011) for a recent tutorial.

This paper revisits the modified communication channel from the perspective of time-delay sys-

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tems theory using spectral methods considering first-order linear passive systems. The problem is motivated by the wide variety of industrial processes described by first-order plants with time-delay and commonly regulated by PI controllers (Silva, Datta, & Bhattacharyya, 2001), such as DC servomotors extensively used in industry. In this paper, the emphasis is put on the performance of the closed-loop system when a communication channel stands between the plant and the controller, which is quantified by its σ-stability degree. Here, σ approximates the exponential decay rate of the system response by means of its rightmost roots (Ramírez, Sipahi, Mondié, & Garrido, accepted).

Our analysis is based on classical results of time-delay systems of retarded and neutral nature (Bellman & Cooke, 1963; Hale & Lunel, 1993). The problem under consideration, though infinite dimensional, involves a reduced number of parameters. Hence, a comprehensive frequency domain analysis of the closed-loop characteristic quasipolynomial can be performed. Particularly, we deploy a critical extension of the $\mathcal{D}$-subdivision method of Neĭmark (1949), see also Sipahi, Niculescu, Abdallah, Michiels, and Gu (2011) for advanced methods, which consists on (i) the determination of the stability boundaries corresponding to roots at $s = -\sigma$ and $s = -\sigma + j\omega$, which provides a partition of the space of PI parameters and (ii) the verification of the relative stability degree σ of each region in the partition. Having generated the complete set of σ-stability boundaries and determined the σ-stability regions, the exact σ-stability maps follow. Finally, based on the idea of assigning a closed-loop transfer triple rightmost root, a topic that has received significant attention in recent theoretical contributions and applications (Boussaada, Morărescu, & Niculescu, 2015; Li, Niculescu, Cela, Wang, & Cai, 2013; Michiels & Niculescu, 2007; Ramírez et al., accepted; Zítek, Fišer, & Vyhlídal, 2013), a fully analytic characterization of the maximum achievable exponential decay rate resulting in simple tuning formulae for practitioners, is provided.

The contribution is organized as follows: In Section 2, the delay-free and the fixed, non-zero delay cases are analyzed, illustrating the failure and loss of performance in the passivity-based design strategy. In Section 3, the scattering transformation is introduced and the characteristic function is obtained, the σ-stability maps are sketched and tuning rules for points of interest are derived. A theoretical limit on the σ-stability is found and a tuning rule for the scattering transformation is proposed. It is shown that, when the rule is followed, the theoretical limit can be approached arbitrarily close. Concluding remarks are given in Section 4.

2. Problem statement

Consider a first-order linear system of the form

$$\dot{x} = -ax + bu_1 \ , \tag{1a}$$
$$y_1 = x \ , \tag{1b}$$

where u_1, y_1 and $x \in \mathbb{R}$ are the input, output and state, respectively. The parameters a and b are assumed to be non negative, which ensures passivity with storage function $V_1(x) = x^2/(2b)$.

Consider the PI controller

$$\dot{\xi} = u_0 \ , \tag{2a}$$
$$y_0 = k_p u_0 + k_i \xi \ , \tag{2b}$$

where u_0, y_0, $\xi \in \mathbb{R}$ are the controller input, output and state, respectively. The proportional and integral gains k_p and k_i are assumed to be positive, so the controller is also passive. The system and the controller are interconnected as per the following pattern

$$u_1 = -y_0 \quad \text{and} \quad u_0 = y_1 - y_1^{\star}.$$

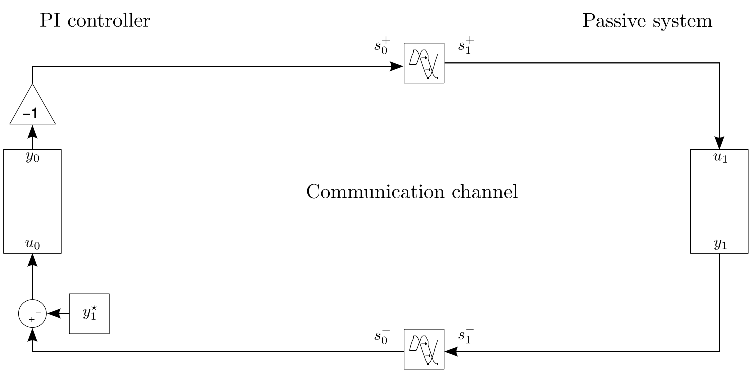


Figure 1. A control-loop including a delayed communication channel.

Since the interconnection of two passive systems is again passive, the closed-loop characteristic polynomial is stable for all gains.

However, when a communication channel with delays is introduced in the loop, as shown in Fig. 1, the closed-loop transfer function takes the form

$$\frac{y_1(s)}{y_1^*(s)} = \frac{(k_p s + k_i) b e^{-h_1 s}}{s^2 + as + (k_p s + k_i) b e^{-hs}} \tag{3}$$

with h_1 the forward delay, h_2 the return delay and $h = h_1 + h_2$ the round-trip delay. The stability properties of the closed-loop are then defined by the location of the roots of the characteristic equation

$$p(s, k_p, k_i) = s^2 + as + (k_p s + k_i) b e^{-hs} , \tag{4}$$

also known as the characteristic quasipolynomial. Notice that the presence of the delay $h > 0$ in the communication channel induces infinite-dimensionality to the system due to the exponential term and therefore the quasipolynomial in (4) bears an infinite number of roots. Since it is impossible to compute all these roots, the stabilization of the zero-solution is not trivial. Moreover, since the delay channel is not passive, the passivity argument fails and stability can no longer be ensured for every combination of positive parameters.

In the following we consider the problem of finding the setting on the parameters (k_p, k_i) that create the maximum decay rate for the system (1)-(2) in the presence/absence of time-delays.

2.1 *Performance degradation as a result of delays*

As a preliminary step for the characterization of the maximum decay rate, we begin with the decomposition of the (k_p, k_i)-plane by using the $\mathcal{D}$-subdivision method, which states that stability switches happen only if at least one characteristic root crosses the imaginary axis. Therefore, we have to search for the crossing points (k_p, k_i) and the corresponding crossing frequency ω such that

$$p(j\omega, k_p, k_i) = 0. \tag{5}$$

Due to symmetry of the characteristic roots with respect to the real axis, we may consider only non-negative frequencies; i.e, $\omega \in [0, \infty)$. Besides pure stability, we will be concerned with the exponential decay of system solutions with a given degree σ, that is, with the σ-stability of the system. This happens only if all the roots of the characteristic quasipolynomial have real parts less than $-\sigma$ (Gu, Kharitonov, & Chen, 2003). With this in mind, we now have to search for stability switching along the imaginary axis shifted a distance equal to $-\sigma$. Then, we will determine the set of all (k_p, k_i) points for which the closed-loop system is σ-stable.

The first step of the $\mathcal{D}$-subdivision method requires to equate (4) to zero and set $s = -\sigma + j\omega$, which gives $p(-\sigma + j\omega, k_p, k_i) = p(\sigma, \omega, k_p, k_i) = 0$, where

$$p(\sigma, \omega, k_p, k_i) = (-\sigma + j\omega)^2 + a(-\sigma + j\omega) + \big(k_p(-\sigma + j\omega) + k_i\big)be^{-h(-\sigma+j\omega)} = 0\,. \tag{6}$$

Notice here that σ and ω appear non-linearly. The gains k_p and k_i, however, appear affinely. Then, finding the crossing points (k_p, k_i) requires collecting the real and the imaginary parts of (6) and solving the resulting equations for k_p and k_i. Following this procedure results in the set of parametric equations

$$k_p(\omega) = \frac{-a_{2\sigma}\omega\cos(h\omega) + \big(\sigma a_\sigma + \omega^2\big)\sin(h\omega)}{b\omega e^{h\sigma}}\,, \tag{7a}$$

$$k_i(\omega) = \frac{(\sigma^2 + \omega^2)\,(\omega\cos(h\omega) + a_\sigma\sin(h\omega))}{b\omega e^{h\sigma}}, \tag{7b}$$

where, for notational simplicity, we have defined $a_\sigma = a - \sigma$ and $a_{2\sigma} = a - 2\sigma$. Then, with a fixed σ and sweeping $\omega \in (0, \infty)$ allows the obtention of all k_p and k_i that generates a root on the shifted imaginary axis. Also, observe that we have intentionally excluded the value $\omega = 0$ in the sweeping process to avoid any indetermination. This case is treated separately performing the second step of the $\mathcal{D}$-subdivision method, which follows the same logic above. Hence, we equate (4) to zero, set $s = -\sigma + j0$, and obtain $p(-\sigma + j0, k_p, k_i) = p(\sigma, 0, k_p, k_i) = 0$, where

$$p(\sigma, 0, k_p, k_i) = (-\sigma)^2 + a(-\sigma) + \big(k_p(-\sigma) + k_i\big)be^{-h(-\sigma)} = 0\,.$$

Solving the above equation for k_i gives the boundary

$$k_i(0) = \sigma k_p + \frac{\sigma a_\sigma}{be^{h\sigma}}. \tag{8}$$

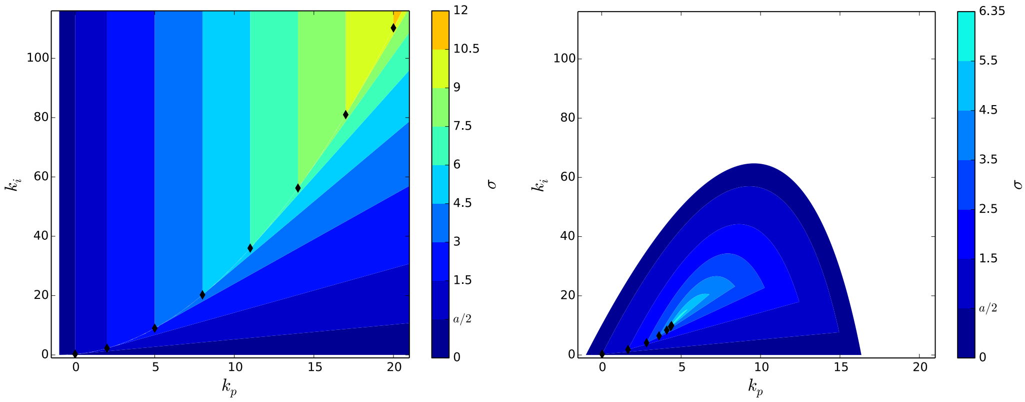
Equipped with (7), (8) and under continuity arguments, we can now declare the exact regions in (k_p, k_i)-plane for which the quasipolynomial (4) is σ-stable. We will refer to the collection of all the crossing points obtained from (7), (8) as the stability crossing boundaries.

Remark on non-dimensionalization: In investigating the spectral properties of the considered control-loop it is not necessary to distinguish between every possible combination of (a, b) parameters. In fact, introducing the scaling variables $\tau = at$ and $u_1' = (b/a)u_1$, the non-dimensional form of the general plant (1) is obtained as

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\tau}x(\tau) &= -x(\tau) + u_1'(\tau)\,,\\ y_1(\tau) &= x(\tau)\,,\end{aligned}$$

where the dependence on (a, b) is obviated. In other words, without loss of generality we can set $a = b = 1$ in the figures that follow for demonstration of the control-loop properties. In consequence, and consistent with the above non-dimensionalization, the figures for all possible (a, b) combinations might change quantitatively, but the qualitative features remain unchanged.

Considering that σ is given, equations (7), (8) decompose the (k_p, k_i)-plane into a finite number of disjoint regions. Due to continuity arguments, each of these regions is then characterized by



(a) Delay-free case, $h = 0$. Achievable decay is unbounded.

(b) With delays, $h = 0.1$. Maximal achievable decay is $\sigma^* = 6.35$.

Figure 2. σ-stability regions. Communication channel without scattering transformation. The diamond markers correspond to minimal gains ensuring a given σ.

the same number of strictly σ-unstable characteristic roots. We will refer to the collection of all pairs (k_p, k_i) for which the number of σ-unstable roots of $p(s, k_p, k_i)$ in (4) is zero as the σ-stability domain $\mathcal{D}_\sigma$.

Using $a = b = 1$ in (7), (8), we obtain Fig. 2 for $h \geq 0$. In order to assist the reader, for a given σ the corresponding $\mathcal{D}_\sigma$ is filled in color and the stability crossing boundaries are trimmed to fit the boundary of $\mathcal{D}_\sigma$. Two interesting observations are in order. Firstly, in Fig. 2(b) one can observe that searching for a faster response will eventually result in the collapse of $\mathcal{D}_\sigma$. We have that, at the critical value σ^* where the boundaries obtained from (7), (8) meet each other at $\mathcal{D}_{\sigma^*}$, a triple real dominant root will be generated at $-\sigma^*$. This point corresponds to the maximum achievable closed-loop exponential decay rate σ^*, and these arguments are consistent with previous discussions on the maximum achievable σ, see (Michiels & Niculescu, 2007) Theorem 7.6. Secondly, from Fig. 2(a), in contrast with the delayed case, it is clear that achieving an arbitrarily large exponential decay σ is possible in the absence of communication delays.

Let us characterize the maximal achievable decay σ^* for the delayed case. This characterization is then particularized to conclude on the relative stability properties of the delay-free case.

Proposition 1: *Consider a plant (1) in closed-loop with a PI controller (2) satisfying $k_p \geq 0$ and $k_i \geq 0$. A delayed communication channel with round-trip delay $h > 0$ stands between the system and the controller.*

(i) The maximal achievable exponential decay is given by

$$\sigma^* = \frac{4 + ah - \sqrt{8 + a^2h^2}}{2h} \,. \tag{9}$$

(ii) The minimal PI controller gains assigning a given $\sigma^* \geq \sigma \geq a/2$ are

$$k_p = \frac{\sigma h a_\sigma - a_{2\sigma}}{b e^{h\sigma}} \,, \tag{10a}$$

$$k_i = \frac{\sigma^2 (h a_\sigma + 1)}{b e^{h\sigma}} \,, \tag{10b}$$

where condition $\sigma \geq a/2$ ensures that $k_p \geq 0$ for all $h > 0$.

Proof. (i) For a given σ, the corresponding stability domain $\mathcal{D}_\sigma$ is delimited by the parametric equations corresponding to a real root at $s = -\sigma$ and a pair of complex roots $s = -\sigma \pm j\omega$ of the quasipolynomial (4). Continuously increasing σ creates a displacement of the stability crossing boundaries, where σ-stability is lost after $\mathcal{D}_\sigma$ collapses (Ramírez et al., accepted). As per the smoothness of the crossing boundaries (Mendez-Barrios, Niculescu, Morarescu, & Gu, 2008), the collapse point is characterized by a triple real root[1] at $-\sigma$. Thus, the quasipolynomial (4), its first and its second derivatives must vanish at $s = -\sigma$. That is,

$$p(s, k_p, k_i)\Big|_{s=-\sigma} = 0, \quad p'(s, k_p, k_i)\Big|_{s=-\sigma} = 0, \quad p''(s, k_p, k_i)\Big|_{s=-\sigma} = 0 \tag{11}$$

where p' and p'' are the first and second derivative of p with respect to s. From the above, it follows that

$$be^{h\sigma}(k_i - \sigma k_p) = \sigma a_\sigma \ , \tag{12a}$$

$$-hbe^{h\sigma}(k_i - \sigma k_p) + be^{h\sigma} k_p = -a_{2\sigma} \ , \tag{12b}$$

$$h^2 be^{h\sigma}(k_i - \sigma k_p) - 2hbe^{h\sigma} k_p = -2 \ . \tag{12c}$$

Using these equations, one can eliminate the variables k_p and k_i as follows. First, substituting (12a) into (12b) and (12c) yields

$$-h\sigma a_\sigma + be^{h\sigma} k_p = -a_{2\sigma} \ , \tag{13a}$$

$$h^2 \sigma a_\sigma - 2hbe^{h\sigma} k_p = -2 \ . \tag{13b}$$

Using (13a) and (13b) produce $h^2\sigma^2 - (ha + 4)h\sigma + 2 + 2ah = 0$, whose solutions are

$$\sigma_{1,2} = \frac{4 + ah \pm \sqrt{8 + a^2h^2}}{2h} \ .$$

The σ solution that ensures $k_p \geq 0$ and $k_i \geq 0$ is (9).

(ii) The minimal gains occur at the intersection of the two boundaries. Hence, the conditions (12a) and (12b) must hold. The result follows by solving these equations for k_p and k_i.

□

We end this section with a comment on the performance degradation as a result of delays. To this end, let $h \to 0$ in (9). It follows that the maximum achievable decay rate tends to infinite as the delay vanishes. From a practical point of view, an upper bound for σ^* is solely determined by the physical limitations of the considered delay-free system. Moreover the minimal PI controller gains assigning given $\sigma \geq a/2$ are given by k_p in (10a) and by k_i in (10b) with $h = 0$.

Example 1: Suppose that the plant (1) has parameters $a = b = 1$ and that the loop is closed with a PI controller (2) using a perfect communication channel, i.e., with $h = 0$. We wish to regulate the output to the reference $y_1^\star = 1$ with a desired exponential decay of $\sigma = 10$. According to (10), the minimal gains achieving such performance are $k_p = 19$ and $k_i = 100$. The expected well-behaved response is shown in Fig. 3 (solid black).

Suppose now that a delay $h = 0.1$ is introduced in the communication channel. According to Fig. 2.1 (right hand), the system is unstable for this set of parameters, as the chosen k_p and k_i fall in the white area, which corresponds to a negative exponential decay. The expected unstable behavior is portrayed in Fig. 3 (dotted red).

[1] A formal proof of this fact follows from the analysis of $\partial\sigma/\partial\omega = 0$ using the implicit function theorem, the reader is referred to (Ramírez, Garrido, & Mondié, 2015) for further details in the case of delay-based control of first-order systems.

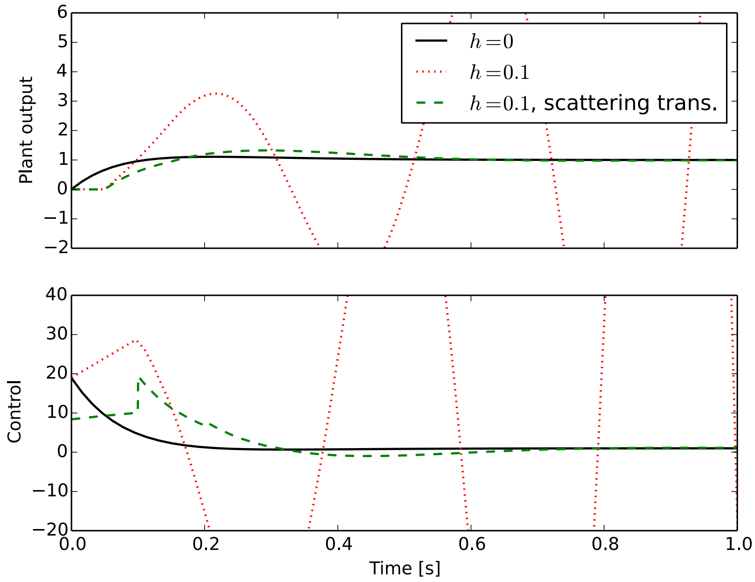


Figure 3. Simulations for the closed-loop (1)-(2) with $a = b = 1$, $k_p = 19$, $k_i = 100$ and $h = 0$ (solid black). Same system as before, but a delay $h = 0.1$ (dotted red). Same set-up, $h = 0.1$, but with the scattering transformation, $d = 15$, implemented in the communication channel (dashed green).

Finally the rest of the paper is dedicated to the problems of (i) improving the relative stability of the system when challenged by the presence of communication delays, (ii) recovering the passive properties of the overall control-loop and (iii) algebraically designing the controller gains to prescribe a desired exponential decay rate.

3. The scattering transformation

A classical approach to the study of transmission lines consists in applying a linear transformation on the state variables (Cheng, 1992). By applying such transformation, the transmission line equations, i.e., the telegrapher's equations, transform into a pair of decoupled delay equations, i.e., transport PDE. It is then possible to understand the dynamics of the transmission line in terms of wave propagation.

The converse was proposed by Anderson and Spong (1989): Suppose we have a communication channel consisting of a pair of delays. Apply the inverse transformation to emulate the behaviour of a transmission line. Since transmission lines are passive (lossless), the passivity argument is restored.

More precisely, consider a pair of delays given by the transport PDE, see Krstic (2009) for applications of the transport PDE to backstepping design,

$$\begin{pmatrix} q_l^+(l,t) \\ q_l^-(l,t) \end{pmatrix} = \begin{pmatrix} -h_1 & 0 \\ 0 & h_2 \end{pmatrix} \begin{pmatrix} q_t^+(l,t) \\ q_t^-(l,t) \end{pmatrix} , \tag{14}$$

where $q_l^\pm$ are the partial derivatives of the scattering variables $q^\pm$ with respect to the spatial variable $l \in [0,1]$. Notice that, at the boundaries, the $q^\pm$ solutions satisfy $q^+(1,t) = q^+(0,t-h_1)$ and $q^-(1,t) = q^-(0,t+h_2)$. This is the communication channel. Consider now the linear transformation

$$\begin{pmatrix} \mu(l,t) \\ \upsilon(l,t) \end{pmatrix} = \begin{pmatrix} 1 & d \\ 1 & -d \end{pmatrix}^{-1} \begin{pmatrix} q^+(l,t) \\ q^-(l,t) \end{pmatrix} , \tag{15}$$

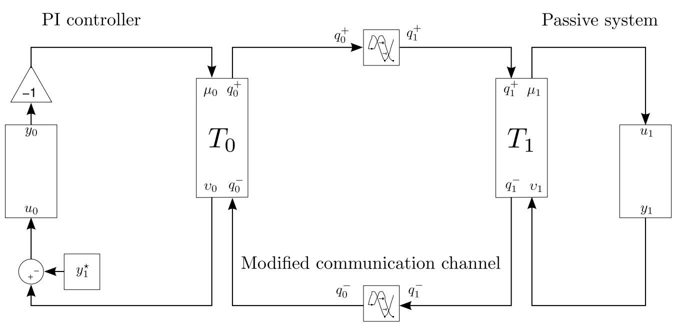


Figure 4. A control-loop including a communication channel with the scattering transformation.

where $d > 0$ is a design parameter. It follows from (15) and (14), with $h = h_1 + h_2$, in general $h_1 \neq h_2$, that

$$\begin{pmatrix} \mu_l(l,t) \\ \upsilon_l(l,t) \end{pmatrix} = -\frac{h}{2} \begin{pmatrix} 0 & d \\ 1/d & 0 \end{pmatrix} \begin{pmatrix} \mu_t(l,t) \\ \upsilon_t(l,t) \end{pmatrix} ,$$

where (μ_l, ν_l) and (μ_t, ν_t) are respectively the spatial and temporal partial derivatives of (μ, ν). The above PDE corresponds to the (lossless) telegrapher's equations.

Transformation (15) can be enforced at the boundaries of (14). However, it is necessary to be cautious in respecting the causality of the system: The variables $q_0^+ := q^+(0,t)$, $q_1^- := q^-(1,t)$, $\mu_0 := \mu(0,t)$ and $\upsilon_1 := \upsilon(1,t)$ are free, while $q_1^+ := q^+(1,t)$, $q_0^- := q^-(0,t)$, $\upsilon_0 := \upsilon(0,t)$ and $\mu_1 := \mu_1(1,t)$ depend on the former variables and their past values. With these restrictions in mind, we can set, at the boundary $l = 0$,

$$\begin{pmatrix} \upsilon_0 \\ q_0^+ \end{pmatrix} = \begin{pmatrix} 1/d & -1/d \\ 2 & -1 \end{pmatrix} \begin{pmatrix} \mu_0 \\ q_0^- \end{pmatrix} , \tag{16a}$$

(cf. (15) at $l = 0$). At $l = 1$ we have

$$\begin{pmatrix} \mu_1 \\ q_1^- \end{pmatrix} = \begin{pmatrix} -d & 1 \\ -2d & 1 \end{pmatrix} \begin{pmatrix} \upsilon_1 \\ q_1^+ \end{pmatrix} , \tag{16b}$$

(cf. (15) at $l = 1$). Here, equation (16) is known as the scattering transformation. Further details can be found in (Niemeyer & Slotine, 1991, 2004; Nuño et al., 2011; Nuño, Basañes, Ortega, & Spong, 2009). Finally, we can use the interconnection pattern

$$u_0 = \upsilon_0 - y_1^\star , \quad \mu_0 = -y_0 , \quad u_1 = \mu_1 \quad \text{and} \quad \upsilon_1 = y_1 , \tag{17}$$

as in Fig. 4. We stress that, since a pair of pure delays is not passive, the stability properties of the system cannot be changed by changing the representation alone. To obtain a passive communication channel, the proposed scattering transformations have to be effectively implemented at the endpoints, so the original system and the one with the scattering transformation are not the same. It is worth noting that the proposed transformations are linear and algebraic, and can be implemented easily in a very concrete fashion.

3.1 *Recovering stability and improving σ-stability*

When the scattering transformations are introduced as shown in Fig. 4, and the parameter d is arbitrary, the closed-loop transfer function is obtained from (1), (2) and (16) and using both the interconnection pattern (17) and the equations of the delayed communication channel $q_1^+ = q_0^+(t-h_1)$ and $q_0^- = q_1^-(t-h_2)$. Then, we have

$$\frac{y_1(s)}{y_1^*(s)} = \frac{2d(k_p s + k_i)be^{-h_1 s}}{p_2(s)s^2 + p_1(s)s + p_0(s)} ,$$

where

$$\begin{aligned}
p_2(s) &= (1+e^{-hs})d + (1-e^{-hs})k_p , \\
p_1(s) &= (1+e^{-hs})(bk_p + a)d + (1-e^{-hs})(bd^2 + ak_p + k_i) , \\
p_0(s) &= (1+e^{-hs})bk_i d + (1-e^{-hs})ak_i .
\end{aligned}$$

The characteristic quasipolynomial is thus

$$p(s) = p_2(s)s^2 + p_1(s)s + p_0(s) . \tag{18}$$

Notice that this quasipolynomial is of neutral type. In other words, its principal coefficient, $p_2(s)$, contains exponential terms, see (Bellman & Cooke, 1963; Kharitonov & Mondié, 2011) for a discussion and analysis of quasipolynomials. In the time domain, this means that the time derivative of the state of the system does not only depend on delayed states, but also on the derivative of the delayed state.

To determine the boundaries of the σ-stability region, we equate the quasipolynomial (18) to zero, set $s = -\sigma$ and solve for k_i as a function of k_p,

$$k_i = \sigma k_p + \sigma \frac{(1+e^{-h\sigma})a_\sigma + (1-e^{-h\sigma})bd}{(1-e^{-h\sigma})a_\sigma + (1+e^{-h\sigma})bd} d . \tag{19}$$

Now, setting $s = -\sigma + j\omega$ and solving for k_p and k_i gives the implicit parametric equations

$$A(\omega)\begin{pmatrix} k_p \\ k_i \end{pmatrix} = d \cdot B(\omega) , \tag{20}$$

where

$$\begin{aligned}
A_{11}(\omega) &= (\gamma\omega - \alpha\sigma)bd - \beta(\sigma a_\sigma + \omega^2) - \gamma\omega a_{2\sigma} , \\
A_{12}(\omega) &= \alpha bd + \beta a_\sigma - \gamma w , \\
A_{21}(\omega) &= (\gamma\sigma + \alpha\omega)bd - \gamma(\sigma a_\sigma + \omega^2) + \beta\omega a_{2\sigma} , \\
A_{22}(\omega) &= -\gamma bd + \gamma a_\sigma + \beta\omega , \\
B_1(\omega) &= (\beta\sigma + \gamma\omega)bd + \alpha(\sigma a_\sigma + \omega^2) - \gamma\omega a_{2\sigma} , \\
B_2(\omega) &= (\gamma\sigma - \beta\omega)bd - \gamma(\sigma a_\sigma + \omega^2) - \alpha\omega a_{2\sigma} .
\end{aligned}$$

and

$$\alpha = 1 + e^{h\sigma}\cos(h\omega) \,,$$
$$\beta = 1 - e^{h\sigma}\cos(h\omega) \,,$$
$$\gamma = e^{h\sigma}\sin(h\omega) \,.$$

Recall that a necessary condition for the stability of neutral-type delay systems is the stability of the difference operator (Hale & Lunel, 1993; Michiels & Niculescu, 2007; Olgac & Sipahi, 2004; Olgac, Vyhlídal, & Sipahi, 2008), which is the inverse Laplace transform of $p_2(s)$. When $\sigma = 0$, the difference equation becomes

$$(d + k_p)\, x(t) + (d - k_p)\, x(t-h) = 0 \,.$$

Since $|(d - k_p)\,/\,(d + k_p)| < 1$, the stability of the difference operator always holds. On the contrary, when $\sigma > 0$, the characteristic equation of the difference operator becomes

$$d + k_p + e^{-hs}e^{h\sigma}(d - k_p) = 0 \,,$$

(which is obtained by making the substitution $s \to s - \sigma$ in $p_2(s)$). The necessity on the stability of the difference operator imposes the new condition

$$e^{h\sigma}\left|\frac{d-k_p}{d+k_p}\right| < 1$$

or, equivalently,

$$d\frac{e^{h\sigma}-1}{e^{h\sigma}+1} < k_p < d\frac{e^{h\sigma}+1}{e^{h\sigma}-1} \,. \tag{21}$$

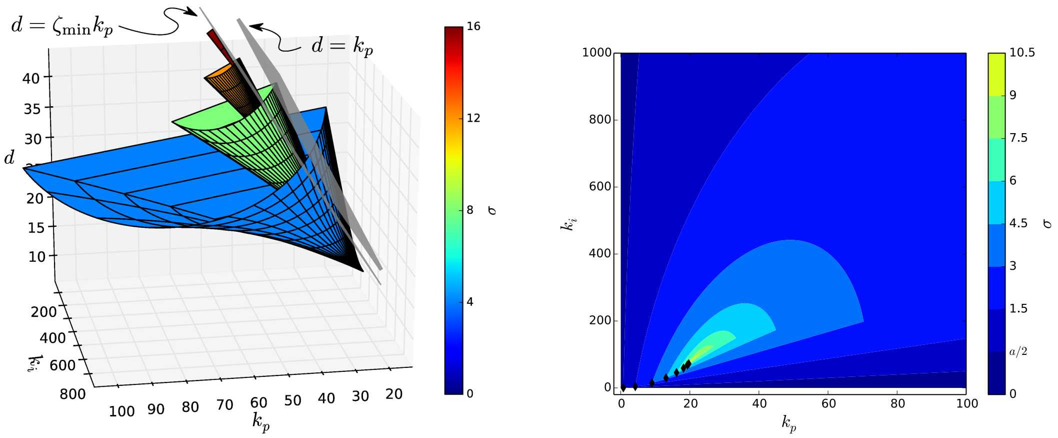
The expressions (19), (20) and (21) are used to determine the σ-stability regions in the (k_p, k_i, d)-space of parameters. These regions are shown in Fig. 5(a). For illustration purposes, the slice $d = 15$ is also shown in Fig. 5(b).

Remark 1: The scattering transformation recovers the striking property, observed in the delay-free case, that the whole first quadrant of the parameter space (k_p, k_i) ensures stability of the closed-loop. Regarding σ-stability, the size of the regions in Fig. 5(b) are larger than those in Fig. 2(b). Also, the maximal achievable exponential decay, σ^*, is greater with the scattering transformation than without it for sufficiently large d.

Example 2: Consider the set-up of Example 1, but suppose that the scattering transformation is implemented in the communication channel (see Fig. 4). For the sake of concreteness, set $d = 15$. According to the previous discussion, stability is recovered. Moreover, according to Fig. 5(b) the exponential decay should be close to the original one, $\sigma \approx 10$. This is confirmed in the simulation shown in Fig. 3 (dashed green).

Again, we characterize the maximal achievable decay and the corresponding control gains.

Proposition 2: *Consider a plant (1) in closed-loop with a PI controller (2) satisfying $k_p \geq 0$ and $k_i \geq 0$. A communication channel with round-trip delay $h > 0$ stands between the system and the controller. For a given $d > 0$, apply the scattering transformations (16) at the channel end points.*



(a) Regions of σ-stability in the (k_p, k_i, d)-space of parameters. The upper parts of the regions have been clipped so that inner regions corresponding to higher σ are visible. The planes $d = k_p$ and $d = \zeta_{\min} k_p$ are shown in light gray.

(b) σ-stability regions on the slice $d = 15$. The diamond markers correspond to the minimal gains that ensure a given σ. Maximal achievable decay is $\sigma^*_{d=15} = 10.9$.

Figure 5. Communication channel with delays, $h = 0.1$, and scattering transformation.

(i) *The maximal achievable decay σ^*_d is a root σ of $m_d(\sigma)$, where*

$$\begin{aligned} m_d(\sigma) = (1 + e^{h\sigma}) \left(2hb^3d^3 + (h^2\sigma a_\sigma + 4)b^2d^2 + 2h(\sigma^2 - a_{2\sigma}^2)bd - h^2\sigma a_\sigma^3\right) \\ + (1 - e^{h\sigma}) \left(h^2\sigma b^3 d^3 + 2h(a + \sigma)b^2d^2 + (4a - h^2\sigma a_\sigma^2)bd - 2ha_\sigma^3\right) \end{aligned} \tag{22}$$

such that the gains

$$k_p = d\frac{(bd - a_\sigma)^2 e^{2h\sigma} + 2\sigma e^{h\sigma}\left(2bd + h(b^2d^2 - a_\sigma^2)\right) - (bd + a_\sigma)^2}{\left(bd + a_\sigma + e^{h\sigma}(bd - a_\sigma)\right)^2}\,, \tag{23a}$$

$$k_i = 2d\sigma^2 e^{h\sigma}\frac{2bd + h(b^2d^2 - a_\sigma^2)}{\left(bd + a_\sigma + e^{h\sigma}(bd - a_\sigma)\right)^2}\,. \tag{23b}$$

are non negative and such that (21) holds.

(ii) *The minimal PI controller gains assigning a given $\sigma^*_d \geq \sigma > a/2$ are given by (23).*

Proof. (i) As in the case where no scattering transformation is employed, the σ-stability regions collapse at a triple root at $s = -\sigma$. The fact that the quasipolynomial (18) and its first derivative are zero give the conditions, written in compact form,

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} k_p \\ k_i \end{pmatrix} = d \cdot \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}\,, \tag{24}$$

where

$$A_{11} = -(1+e^{h\sigma})\sigma bd - (1-e^{h\sigma})\sigma a_\sigma \ ,$$
$$A_{12} = (1+e^{h\sigma})bd + (1-e^{h\sigma})a_\sigma \ ,$$
$$A_{21} = bd + a_{2\sigma} + e^{h\sigma}\left((bd-a_\sigma)(h\sigma+1)+\sigma\right) \ ,$$
$$A_{22} = 1 - e^{h\sigma}(h(bd-a_\sigma)+1) \ ,$$

and

$$B_1 = (1-e^{h\sigma})\sigma bd + (1+e^{h\sigma})\sigma a_\sigma \ ,$$
$$B_2 = -(bd+a_{2\sigma}) + e^{h\sigma}\left((bd-a_\sigma)(h\sigma+1)+\sigma\right) \ .$$

The solution of this linear system of equations is given by (23). Computing the second derivative of (18) gives

$$\begin{pmatrix} A_{31} & A_{32} \end{pmatrix} \begin{pmatrix} k_p \\ k_i \end{pmatrix} = d \cdot B_3 \ , \tag{25}$$

where

$$A_{31} = \Big(2 - e^{h\sigma}\left((h(bd-a_\sigma)+1)(h\sigma+2)+h\sigma\right)\Big) \ ,$$
$$A_{32} = he^{h\sigma}(h(bd-a_\sigma)+2) \ ,$$
$$B_3 = -\Big(e^{h\sigma}\left((h(bd-a_\sigma)+1)(h\sigma+2)+h\sigma\right)+2\Big) \ .$$

Substituting (23) into (25) gives the implicit equation $m_d(\sigma) = 0$. Then, σ_d^* is given as a root of $m_d(\sigma)$ satisfying the non negative condition on the PI control gains and the necessary stability condition (21) associated to the neutral nature of the quasipolynomial.

(ii) The minimal control gains occur at the intersection of two boundaries. The result follows by noting that at this intersection a double root of the quasipolynomial (18) arises, in other words, when (23) holds.

□

Observe that the vanishing of the quasipolynomial (18) and its first two derivatives are necessary conditions for the σ-stability regions to collapse. Therefore, the roots of (22) must be verified via back substitution into the solution of equation (24). Then, if $k_p \geq 0$, $k_i \geq 0$ and (21) hold, the σ_d^*-stability of the detected collapse point has to be checked for σ_d^* to be feasible.

Remark 2: It follows from the implicit function theorem that σ_d^* can also be characterized as the value of σ at which the derivative of (23a) with respect to σ is equal to zero.

3.2 *Least upper bound on the exponential decay rate*

To provide an idea of how restrictive (21) is, we have computed the minimal gains (23) and plotted them in Fig. 6. Notice that, when the minimal gains are used, the upper bound in (21) is never infringed. The restriction $\sigma > a/2$, on the other hand, ensures the lower bound.

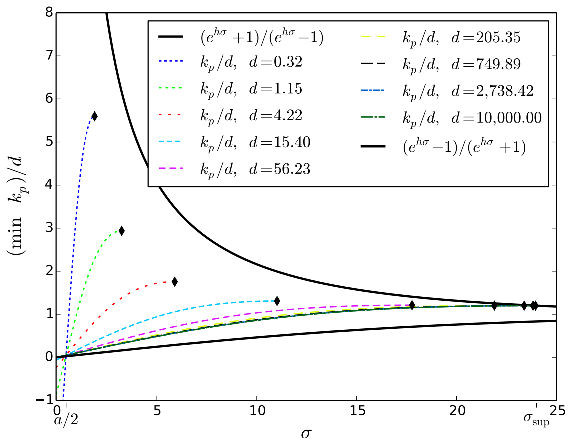


Figure 6. Minimal k_p as a function of σ (23a), normalized by d. The bounds (21) are also shown. The diamond markers correspond to σ_d^*, in accordance with (22). Notice that $k_p(\sigma_d^*)/d$ converges to a constant value as $d \to \infty$.

It is clear from Fig. 6 that, the maximal exponential decays σ_d^* is a monotonically increasing function of d. It is also clear that, as $d \to \infty$ the σ_d^* accumulate at a point, which we denote by σ_{sup}. This is formalized in the following proposition.

Proposition 3: *In the limit, as $d \to \infty$, the maximal exponential decays σ_d^* are bounded by σ_{sup}, which is defined implicitly by $h\sigma_{\text{sup}} = \eta$ with η satisfying*

$$2(1 + e^{\eta}) + \eta(1 - e^{\eta}) = 0\ . \tag{26}$$

Notice that σ_d^* is unbounded for $h = 0$.

Proof. Notice that $m_d(\sigma)$ is asymptotically equivalent to

$$\Big[2(1 + e^{h\sigma}) + h\sigma(1 - e^{h\sigma})\Big]\, hb^3d^3$$

as $d \to \infty$, the roots of which are given by (26) with $h\sigma = \eta$. □

Notice that σ_{sup} is independent of the system parameters: it depends on h only. In our example we have $h = 0.1$, which gives $\sigma_{\text{sup}} = 23.99$ and agrees with Fig. 6.

The notion of *impedance matching*, from the theory of transmission lines, suggests the choice $d = k_p$ in the scattering transformation (16). The rationale behind this choice is that, in a real transmission line, it avoids wave reflections (Niemeyer & Slotine, 2004). From a frequency-domain perspective, the immediate advantage of this choice is that the characteristic quasipolynomial (18) simplifies substantially, as the coefficients become

$$\begin{aligned} p_2(s) &= 2k_p\ , \\ p_1(s) &= 2k_p(a + bk_p) + (1 - e^{-hs})k_i\ , \\ p_0(s) &= (1 + e^{-hs})bk_pk_i + (1 - e^{-hs})ak_i\ . \end{aligned}$$

The principal coefficient becomes constant, i.e., the closed-loop system is of retarded nature instead of neutral, which obviates the need to verify the stability of the difference operator given by the additional constraint (21).

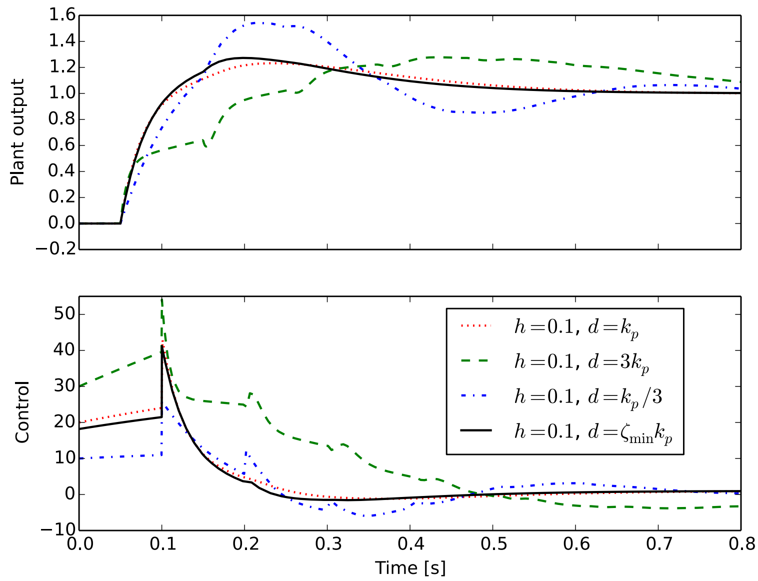


Figure 7. Simulations for the closed-loop (1)-(2) with $a = b = 1$, $k_p = 40$, $k_i = 180$, $h = 0.1$ and scattering transformation with different values of d.

Example 3: Suppose that the plant (1) has parameters $a = b = 1$ and that the controller (2) has parameters $k_p = 40$ and $k_i = 180$. The transformed communication channel has a delay $h = 0.1$ and we set $d = k_p$ in the scattering transformation. The response of the system is shown in Fig. 7. For comparison purposes, we include the simulations resulting from $d = 3k_p$ and $d = k_p/3$. It can be verified that the parameter $d = k_p$ out performs the other two in terms of the convergence velocity towards the reference $y_1^\star$.

The restriction $d = k_p$ corresponds to the plane shown in Fig. 5(a). Notice that, while it is a reasonable choice because of the arguments given above, the plane fails to intersect many σ-stability surfaces. Thus, the choice $d = k_p$ is not optimal in the sense that it obstructs the achievement of σ_{sup}-stability. However, notice from Fig. 6 that the normalized minimal gain, $k_p(\sigma_d^*)/d$, converges to a constant value as $d \to \infty$. This fact suggests the more general linear relation

$$d = \zeta k_p \, , \tag{27}$$

where $\zeta > 0$ is a new design parameter. In the reminder of this section, we will show that there exists a privileged value of ζ, which we call $\zeta_{\min}$ and that optimizes the bound on σ.

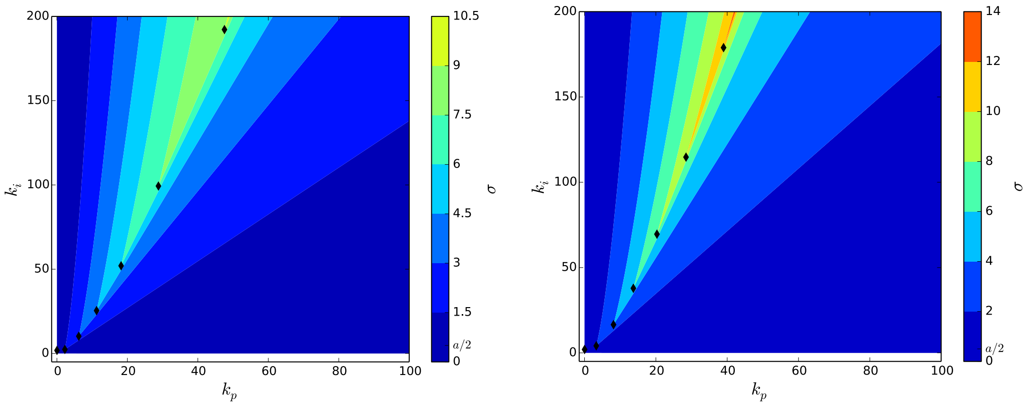
Let us first compute the σ-stability boundaries on the plane given by (27). The boundaries associated to the real roots $s = -\sigma$ are simply given by (19) and (27). On the other hand, solving (20) with $d = \zeta k_p$ is slightly more difficult, since $A(\omega)$ and $B(\omega)$ now depend on k_p. It follows from Cramer's rule that

$$|A(\omega)|\, k_p + d \left| \begin{pmatrix} A_2(\omega) & B(\omega) \end{pmatrix} \right| = 0 \, , \tag{28}$$

where $A_2(\omega)$ stands for the second column of $A(\omega)$ and $|\cdot|$ stands for the determinant. Substituting (27) into (28) gives the equation

$$|A(\omega)| + \zeta \left| \begin{pmatrix} A_2(\omega) & B(\omega) \end{pmatrix} \right| = 0 \, .$$

Developing this equation explicitly leads to the second order polynomial equation $c_2 b^2 \zeta^2 k_p^2 +$



(a) With $\zeta = 1$. The exponential decay is bounded by $\sigma^*_{\zeta=1} = 12.78$.

(b) With $\zeta = \zeta_{\min}$. The exponential decay is bounded by $\sigma^*_{\zeta=\zeta_{\min}} = \sigma_{\sup} = 23.99$.

Figure 8. σ-stability regions on the plane $d = \zeta k_p$. Communication channel with delay, $h = 0.1$, and scattering transformation. The diamond markers correspond to the minimal gains that ensure a given σ.

$c_1 b\zeta k_p + c_0 = 0$ with

$$\begin{aligned}
c_2 &= \Big((\zeta-1)e^{2h\sigma} - (\zeta+1) - 2e^{h\sigma}\cos(h\omega)\Big)\,\omega + 2\zeta e^{h\sigma}\sin(h\omega)\sigma\ ,\\
c_1 &= 2\left(\Big(-(\zeta-1)e^{2h\sigma} - (\zeta+1)\Big)\,a_\sigma + 2e^{h\sigma}\left(\zeta\sigma\cos(h\omega) + \omega\sin(h\sigma)\right)\right)\omega\ ,\\
c_0 &= \left(\omega^2 + a_\sigma^2\right)\left(\Big(e^{2h\sigma}(\zeta-1) - (\zeta+1)\Big)\,\omega + 2e^{2h\sigma}\left(\omega\cos(h\omega) - \zeta\sigma\sin(h\omega)\right)\right)\ .
\end{aligned}$$

The roots of this polynomial can be computed explicitly. Finally, k_i can be computed as

$$k_i = A_{12}^{-1}(\omega)\left(dB_1(\omega) - A_{11}(\omega)k_p\right)$$

with (27).

Fig. 8(a) shows the σ-stability regions for the plane $d = k_p$. Notice that, as in Fig. 2(a), the boundaries are not given by closed curves. This is a clear advantage with respect to the case in which d is constant (cf. Fig. 5(b)). Unfortunately, the choice $d = \zeta k_p$ still imposes a bound on the exponential decay, as it will be shown shortly.

Definition 1: Let $\eta_{\sup}$ be the positive solution of

$$2(1 + e^{\eta_{\sup}}) + \eta_{\sup}(1 - e^{\eta_{\sup}}) = 0 \tag{29}$$

and let $\zeta_{\min}$ be defined as

$$\zeta_{\min} = \frac{(1 + e^{\eta_{\sup}})^2}{2\eta_{\sup}e^{\eta_{\sup}} - (1 + e^{\eta_{\sup}})(1 - e^{\eta_{\sup}})}\ .$$

Proposition 4: *Consider a plant (1) in closed-loop with a PI controller (2) satisfying $k_p \geq 0$ and $k_i \geq 0$. A communication channel with round-trip delay $h > 0$ stands between the system and the controller. Set $d = \zeta k_p$ with $\zeta > 0$ and apply the scattering transformations (16) at the channel end points.*

(i) The least upper bound on the exponential decay is

$$\sigma_\zeta^* = \begin{cases} \frac{1}{h}\ln\left(\frac{1+\zeta}{1-\zeta}\right) & \textit{if} \quad 0 < \zeta < \zeta_{\min} \\ \sigma > 0 \textit{ such that } m_\zeta(h\sigma) = 0 & \textit{if} \quad \zeta_{\min} \le \zeta \end{cases} ,$$

where

$$m_\zeta(\eta) = (1+e^\eta)\left(\zeta(1-e^\eta) + 1 + e^\eta\right) - 2\zeta\eta e^\eta . \tag{30}$$

(ii) The minimal k_p assigning a given $\sigma_\zeta^* > \sigma > a/2$ is a root of the second order polynomial $c_2 b^2\zeta^2 k_p^2 + c_1 b\zeta k_p + c_0$ with

$$\begin{aligned} c_2 &= (1+e^{h\sigma})\left(\zeta(1-e^{h\sigma}) + 1 + e^{h\sigma}\right) - 2\zeta h\sigma e^{h\sigma} , \\ c_1 &= 2(1+e^{h\sigma})\left(\zeta(1+e^{h\sigma}) + 1 - e^{h\sigma}\right) a_\sigma - 4\zeta a e^{h\sigma} , \\ c_0 &= \left((1-e^{h\sigma})\left(\zeta(1+e^{h\sigma}) + 1 - e^{h\sigma}\right) + 2\zeta h\sigma e^{h\sigma}\right) a_\sigma^2 , \end{aligned}$$

while the minimal k_i is given by

$$k_i = \frac{(1+e^{h\sigma})\zeta(bk_p + a_\sigma) + (1-e^{h\sigma})(\zeta^2 bk_p + a_\sigma)}{(1+e^{h\sigma})\zeta bk_p + (1-e^{h\sigma})a_\sigma}\sigma k_p .$$

Proof. The coefficients of (18) take the form

$$\begin{aligned} p_2(s) &= (1+e^{-hs})\zeta k_p + (1-e^{-hs})k_p , \\ p_1(s) &= (1+e^{-hs})(bk_p + a)\zeta k_p + (1-e^{-hs})(b\zeta^2 k_p^2 + ak_p + k_i) , \\ p_0(s) &= (1+e^{-hs})bk_i\zeta k_p + (1-e^{-hs})ak_i . \end{aligned}$$

The formulas for computing the minimal gains are obtained by setting (18) and its derivative equal to zero. This proves (ii).

The maximal exponential decay approaches its maximal value, $\sigma_\zeta^*(\zeta)$, as k_p and k_i tend to infinity, that is, as c_2 approaches 0. Notice that $c_2 = 0$ is equivalent to $m_\zeta(h\sigma) = 0$. For fixed ζ, there exists an $\eta > 0$ such that $m_\zeta(\eta) = 0$ if, and only if,

$$\zeta_{\min} \le \zeta .$$

To see this, set $m_\zeta(\eta) = 0$ and write ζ as a function of η:

$$\zeta = \frac{(1+e^\eta)^2}{2\eta e^\eta - (1+e^\eta)(1-e^\eta)} . \tag{31}$$

To find the lower bound we solve $m_\zeta(\eta) = 0$ and

$$\frac{\mathrm{d}}{\mathrm{d}\eta} m_\zeta(\eta) = -2e^\eta\left[\zeta(e^\eta + \eta + 1)) + 1 + e^\eta\right] = 0$$

simultaneously for η. This gives the implicit equation (29). The lower bound on ζ is finally found by substituting η_{sup} in (31), that is, it is given by $\zeta_{\min}$.

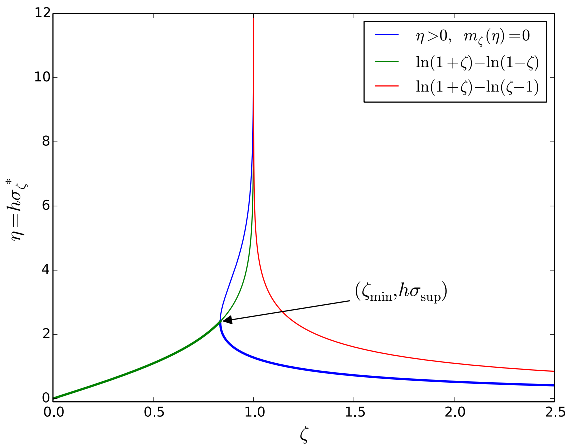


Figure 9. Pairs (η, ζ) solution of $m_\zeta(\eta) = 0$, $\eta > 0$ (in blue). Bounds given by (21) (in green and red). Thick lines correspond to the least upper bound on the exponential decay, multiplied by h.

Thus, $m_\zeta(h\sigma) = 0$ is solvable only if $\zeta_{\min} \leq \zeta$. For $0 < \zeta < \zeta_{\min}$, condition (21) determines the maximal exponential decay. When $d = \zeta k_p$, condition (21) reads

$$h\sigma < \ln\left(\left|\frac{1+\zeta}{1-\zeta}\right|\right) .$$

This proves (i). □

Remark 3: Unlike the bounds given in Propositions 1 and 2, the bound σ^*_ζ is not achievable in practice, as it requires infinite gains. It can, however, be approached arbitrarily close.

The following theorem is important from a practical point of view, since it gives an objective choice for the free parameter in the scattering transformation.

Theorem 1: *Consider a plant (1) in closed-loop with a PI controller (2) satisfying $k_p \geq 0$ and $k_i \geq 0$. A communication channel with round-trip delay $h > 0$ stands between the system and the controller. Set $d = \zeta_{\min} k_p$ and apply the scattering transformations (16) at the channel end points. The least upper bound on σ, $\sigma^*_{\zeta=\zeta_{\min}}$, is equal to $\sigma_{\sup}$.*

Proof. To compute $\zeta_{\min}$ we first solve (29), which is equivalent to (26) with $\eta = h\sigma_{\sup}$. □

Remark 4: The theoretical limit on the exponential decay is independent of the plant parameters. It depends only on the delay h and it is given by

$$\sigma_{\sup} = \frac{\eta_{\sup}}{h} = \frac{2.3994}{h} .$$

The optimal parameter $\zeta_{\min} = 0.8336$, for which σ can be set arbitrarily close to $\sigma_{\sup}$, is independent of the plant parameters and the delay.

Example 4: Consider again the setting of Example 3, but now with $d = \zeta_{\min} k_p$. The simulation results are shown in Fig. 7. Note that the convergence of the output to the desired value $y_1^\star = 1$ is the fastest. This is however, at the expense of the nonsmooth control action predicted by the theory of neutral systems.

In order to assist the reader, the necessary stability condition of the difference operator is plotted in Fig. 9 with $d = \zeta k_p$ along with the solutions of equation $m_\zeta(\eta) = 0$ in (30). As it results from the above analysis, ζ is indeed bounded from below by ζ_{min}. Following this observation, the stability of the difference operator is always satisfied in the interval $\eta_{\sup}/h > \sigma > 0$ and therefore, the σ-stability of the overall control-loop is finally established.

The σ-stability regions on the plane $d = \zeta_{\min} k_p$ are shown in Fig. 8(b) (compare with Fig. 8(a)). Notice that $\sigma^*_{\zeta=\zeta_{\min}}$ is almost twice as large as $\sigma^*_{\zeta=1}$, the value obtained using the recipe usually found in the literature. The plane is also shown in Fig. 5(a).

In view of the preceding remarks we propose the following design procedure:

(i) Set $d = \zeta_{\min} k_p$. This choice is universal, in the sense that it does not depend explicitly on the plant parameters nor the delay.

(ii) Choose a desired σ such that $\eta_{\sup}/h > \sigma > 0$. This choice requires knowledge of the round-trip delay only. The choice of σ should take into account the limits imposed by the actuator, i.e., large σ may generate saturation.

(iii) Use the corresponding minimal gains, as in Proposition 4. This choice requires knowledge of the round-trip delay and the plant parameters.

4. Concluding Remarks

A simple instance of the use of the scattering transformation in passivity-based control has been analyzed from the classical (i.e., frequency-domain) perspective of time-delay systems. The exponential decay rate of the closed-loop system was chosen as a criterion to asses the performance of different control schemes. This criterion leads to an optimal choice on the design parameter of the scattering transformation. Quite remarkably, the optimal choice is independent of the plant parameters and the delays in the communication channel. With the optimal choice, it is possible to attain exponential decay ratios that are almost twice as large as those obtained by setting the design parameter at the value suggested in the literature.

A theoretical limit on the exponential decay rate has been found. The limit does depend on the total delay but, quite remarkably as well, it is independent of the plant parameters.

Distinct qualitative features, such as the shape and extension of the σ-stability regions or the neutral and retarded nature, have been identified for different scenarios: a closed-loop system without delays, with delays, with and without scattering transformation. This furthers our insight on the effect of delays and the scattering transformation used to remedy it.

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