2019 International Conference on Unmanned Aircraft Systems (ICUAS)
Atlanta, GA, USA, June 11-14, 2019

Backstepping-Based Controller for Flight Formation

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Abstract— **A Backstepping controller based on $SE(3)$ for achieving multi-agents consensus and flight formation of a drones fleet is developed in this paper. The controller is obtained using the nonlinear model of the quadrotor and derived with virtual inputs to converge the fleet to desired references. The stability analysis of the controller is analyzed and proved with the Lyapunov theory. Emulations of the control algorithm are carried out for validating the well performance of the closed-loop system.**

I. Introduction

The UAVs (Unmanned Aerial Vehicles) are becoming popular because their ability to perform a variety of missions such as monitoring of civil and military areas, also monitoring different kind of natural disasters or the inspection of any dangerous environments, in the agriculture the UAVs are used to inspect the irrigation areas.

Quadrotors are special class among different types of rotorcrafts. Quadrotor vehicles have gained a lot of research interest due to the clear advantages posed by their vertical take-off and landing, hovering capability and slow precise movements. The vehicle consists of four rotors in total, with two pairs of counter-rotating, fixed-pitch blades located at the four corners of the aircraft. The quadrotor control is involved in stabilize attitude (roll, pitch and yaw) and position (*x, y, z*) either separately or in a coupled way. Due to the unique body structure of a rotorcraft, as well as the rotor dynamics, the rotorcraft attitude dynamics and position dynamics are strongly coupled. Therefore, it is very difficult to design a decoupled control law of good structure that stabilizes the faster and slower dynamics simultaneously [15].

The dynamic model of a quadrotor includes under actuation, strong coupling multi-input and multi-output design, and unknown nonlinearities. In the quadrotor the movement is caused by the resultant forces and moments of four independent rotors. The quadrotor dynamics involve four input forces, six output coordinators and have highly coupled and unstable nonlinear and time varying dynamics, even though many aerodynamic effects are simplified or neglected, hence, the control and the mathematical model to achieve a good performance and autonomous flight is a difficult task.

Different applications for this kind of vehicles could be performed only with only one of them nevertheless multiple vehicles will be more flexible and efficient in performing the tasks. However, using multiple vehicles involve other problems such as the communication and computational power.

The development of powerful control techniques for single vehicles, the explosion in computation and communication capabilities and the advent of miniaturization technologies have elevated interest in vehicles which can interact autonomously with the environment and other vehicles to perform, in the presence of uncertainties and adversities, task beyond the ability of individual vehicles [7].

The cooperative control of UAVs can be defined as a group of drones, that could be of the same dynamics or different, sharing the same objectives to ensure the mission execution successfully. This group of drones can perform the mission keeping a relative distance or specific positions.

Cooperative control and multi-agent robotics are active research areas both in control theory and robotics. Problems such as flocking, consensus, coverage and pattern formation are some of the important problems that have been studied over the past few years.

In general there are four methods to achieve the cooperative control, the behavioral approach is to set different behaviors for each agent in the group while the control of each agent is the weighted average of the control of each behavior. Systems using this technique are able to navigate to way points, avoid obstacles and to keep a desired formation, all at the same time [9].

The main idea of using the graph theory is to use the matrices related to the graph which leads the problem into a linear system analysis with a matrix known as Laplacian that contains all the interactions between agents, hence the problem is reduced to analyzing the eigenvalues of the Laplacian to demonstrate the stability of the formation [1].

The other approach is the virtual structure, in which the main problem is to design the desired dynamics of a virtual structure (in this particular case could be the formation requested) and then just design a control law enough robust to track the desired structure and keep the formation [12], the problem with this methodology is the computation cost of generating the virtual structure.

And the last approach is the leader-follower in which there is an agent that is the leader and the rest of the fleet are the followers, in this approach the leader do not care about the information of the other agents, the problem with this

This work is supported by CONACYT and CINVESTAV, Mexico.

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methodology is not robust against any fault of the leader, to avid this problem, many ideas has been proposed such as the virtual leader which is a virtual particle with the dynamics of the other agents and all the fleet becomes a follower. The advantage of the leader-follower approach is that is relatively easy to implement.

The outline of the paper is the following: the problem statement is described in section II. The mathematical equations of the quadrotor and the attitude control algorithm is introduced in section III. The multi-agent system with its control algorithm for achieving flight formation is explained in section IV. Numerical simulations are realized for validating the performance of the drones fleet, some graphs depicting these results are presented in section V. Finally in section VI the conclusions about this work are discussed.

II. PROBLEM STATEMENT

Formation control of autonomous vehicles is a real challenge addressed in the context of multi-agent systems. In this work, our interest is to give an easy solution for solving the control problem of n quadrotor. In this study, the agents are physically decoupled, but they are interacting between them and exchanging information via wireless communication links.

The main idea of the consensus algorithms is to impose similar (mostly) dynamics behavior to each agent. In this work, the information exchange or interaction between agents is considered piece-wise continuous, so that the evolution can be modeled with differential equations. In addition, fixed and switching topologies are studied with the characteristic that all of them belong to the type of strongly connected graphs. Likewise, the graphs considered are directed.

Therefore, it will be considered that the agents fleet has achieved the consensus if for any initial conditions $x_i(0)$ the next condition fills

$$\lim_{t\to\infty} ||x_i(t) - x_i(t)|| \to 0 \tag{1}$$

and for formation there exists an offset for each agent and mathematically can be expressed as

$$\lim_{t\to\infty} ||x_i(t) - x_i(t)|| \to \Delta_{ij} \tag{2}$$

where x_i and x_j define the states information of the agent i and j respectively and Δ_{ij} represents the relative distance between these both agents, for all $i, j : 1, ..., n$.

The goal of this work is to propose a nonlinear controller that includes the protocol consensus of first order to achieve flight formation and switching topology in aerial vehicle with second order dynamics.

III. QUADROTOR DYNAMICS AND ATTITUDE STABILIZATION

The following nonlinear dynamic model of the quadrotor expressed from Newton-Euler approach is considered

$$\begin{aligned}
\ddot{x} &= -\sin\theta \frac{1}{m} U_1 \\
\ddot{y} &= \cos\theta \sin\phi \frac{1}{m} U_1 \\
\ddot{z} &= \cos\theta \cos\phi \frac{1}{m} U_1 - g \\
\ddot{\phi} &= \dot{\theta}\dot{\phi} \frac{I_y - I_z}{I_x} - \frac{I_r}{I_x}\dot{\theta}\Omega + \frac{l}{I_x} U_2 \\
\ddot{\theta} &= \dot{\phi}\dot{\psi} \frac{I_z - I_x}{I_y} - \frac{I_r}{I_y}\dot{\phi}\Omega + \frac{l}{I_y} U_3 \\
\ddot{\psi} &= \dot{\theta}\dot{\psi} \frac{I_x - I_y}{I_z} + \frac{l}{I_z} U_4
\end{aligned} \tag{3}$$

where (x, y, z) denote the relative distance between the center of mass of the quadrotor and the inertial frame, ϕ, θ, ψ represent the Euler angles used to express the attitude of the vehicle, I_x, I_y, I_z are the inertia along the respective axis, the assumption that the quadrotor is geometrically symmetric is done, the gravity is represented by g and the mass of the vehicle is given by m. This vehicle is a under-actuated system that has more degree of freedoms than control inputs, in this case, four control inputs are used, $U_1, ..., U_4$, from which U_1 is used for the altitude control while the other three are for the attitude stabilization and horizontal displacements.

For control purposes, the aerial vehicle model has to be described in states variables, defining the state vector as $\mathbb{X} = [x_1, x_2, y_1, y_2, z_1, z_2, \phi_1, \phi_2, \theta_1, \theta_2, \psi_1, \psi_2]$ such that

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{y}_1 \\ \dot{y}_2 \\ \dot{z}_1 \\ \dot{z}_2 \\ \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\psi}_1 \\ \dot{\psi}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\sin\theta_1 \frac{1}{m} U_1 \\ y_2 \\ \cos\theta_1 \sin\phi_1 \frac{1}{m} U_1 \\ z_2 \\ \cos\theta_1 \cos\phi_1 \frac{1}{m} U_1 - g \\ \phi_2 \\ \theta_2\phi_2 \frac{I_y - I_z}{I_x} - \frac{I_r}{I_x}\theta_2\Omega + \frac{l}{I_x} U_2 \\ \theta_2 \\ \phi_2\psi_2 \frac{I_z - I_x}{I_y} - \frac{I_r}{I_y}\phi_2\Omega + \frac{l}{I_y} U_3 \\ \psi_2 \\ \theta_2\psi_2 \frac{I_x - I_y}{I_z} + \frac{l}{I_z} U_4 \end{bmatrix} \tag{4}$$

Attitude stabilization

The attitude control of the quadrotor is obtained via the Backstepping approach. System (4) can be studied in many subsystems with second order dynamics with the next form

$$\begin{aligned}
\dot{\eta}_1 &= \dot{\eta}_2 \\
\dot{\eta}_2 &= g(\omega, \gamma) + h(\omega, \gamma) u
\end{aligned} \tag{5}$$

where $g(\omega, \gamma)$, $h(\omega, \gamma)$ are known nonlinear functions and u denotes the control input.

For tracking purposes, it is necessary to stabilize the state in a desired value η_d. Then, the following tracking error can be proposed as $\bar{e}_1 = \eta_1^d - \eta_1$, thus, $\dot{\bar{e}}_1 = \dot{\eta}_1^d - \eta_2$.

Define a positive function and its time derivative as

$$V(\bar{e}_1) = \frac{1}{2}\bar{e}_1^2 \tag{6}$$

$$\dot{V}(\bar{e}_1) = \bar{e}_1(\dot{\eta}_1^d - \eta_2) \tag{7}$$

the state η_2 is defined as a virtual control input such that $\eta_2 \rightarrow \eta_1^d = \dot{\eta}_1^d + \alpha_1\bar{e}_1$, where $\alpha_1 > 0$ is a constant. Introducing the previous into (7) it can be assured that at least the state η_1 is stable.

Define the velocity error as $\bar{e}_2 = \eta_2 - \eta_2^v$ and propose the following candidate Lyapunov function as

$$V_2(\bar{e}_1, \bar{e}_2) = V_1(\bar{e}_1) + \frac{1}{2}\bar{e}_2^2. \tag{8}$$

Then,

$$\begin{aligned}\dot{V}(\bar{e}_1, \bar{e}_2) =& \bar{e}_1(-\alpha_1\bar{e}_1 - \bar{e}_2) + \bar{e}_2\big(g(\omega,\gamma) + \\ & h(\omega,\gamma)u + \alpha_1(\bar{e}_2 + \alpha_1\bar{e}_1)\big)\end{aligned} \tag{9}$$

Hence, the controller yields

$$u = \frac{1}{h(\omega,\gamma)}\Big[-\alpha_1(\bar{e}_2 + \alpha_1\bar{e}_1) - g(\omega,\gamma) + \bar{e}_1 - \alpha_2\bar{e}_2\Big] \tag{10}$$

where α_i is a positive constant. Note that it is necessary to ensure that $h(\omega,\gamma) \neq 0$ to avoid indetermination of the control law. In closed loop the derivate of the Lyapunov function becomes

$$\dot{V}_2 = -\alpha_1\bar{e}_1^2 - \alpha_2\bar{e}_2^2 \leq 0 \tag{11}$$

This impies that the subsystem (5) with the controller (10) has an asymptotically stable behavior.

The consensus algorithm for the drones fleet is considered in the (x,y) plane with a constant altitude z_d, therefore the altitude controller can be obtained using (10), thus it becomes

$$U_1 = \frac{1}{\cos\theta_1\cos\phi_1}\left[g - \alpha_7(e_8 + \alpha_4 e_7) + e_7 - \alpha_8 e_8\right] \tag{12}$$

with $e_7 = z_{1_d} - z_1$ and $e_8 = z_2 - z_2^v$.

Similarly, applying (10) for the attitude dynamics of the quadrotor, the following controllers are obtained

$$\begin{aligned}U_2 =& \frac{I_x}{l}\Big[-\alpha_1(e_2 + \alpha_1 e_1) - \theta_2\phi_2\left(\frac{I_y - I_z}{I_x}\right) + \frac{I_r}{Ix}\theta_2\Omega + \\ & e_1 - \alpha_2 e_2\Big] \\ U_3 =& \frac{I_y}{l}\Big[-\alpha_3(e_4 + \alpha_3 e_3) - \phi_2\psi_2\left(\frac{I_z - I_x}{I_y}\right) + \frac{I_r}{I_y}\phi_2\Omega + \\ & e_3 - \alpha_4 e_4\Big] \\ U_4 =& \frac{I_z}{l}\Big[-\alpha_5(e_6 + \alpha_5 e_5) - \theta_2\psi_2\left(\frac{I_x - I_y}{I_z}\right) + e_5 - \alpha_6 e_6\Big]\end{aligned} \tag{13}$$

with $e_1 = \phi_{1_d} - \phi_1$ and $e_2 = \phi_2 - \phi_2^v$, $e_3 = \theta_{1_d} - \theta_1$ and $e_4 = \theta_2 - \theta_2^v$, $e_5 = \psi_{1_d} - \psi_1$ and $e_6 = \psi_2 - \psi_2^v$.

From the quadrotor flight properties observe that the movements in the plane (x,y) can be achieved using the control inputs U_2 and U_3. This signifies that the position control in this plane can be achieved using the desired angles θ_d and ϕ_d for x and y coordinates respectively. Therefore from (4) the horizontal plane subsystem can be written as

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2 \\ \dot{y}_1 \\ \dot{y}_2\end{bmatrix} = \begin{bmatrix} x_2 \\ -(\sin\theta)\frac{1}{m}U_1 \\ y_2 \\ (\cos\theta\sin\phi\frac{1}{m}U_1\end{bmatrix} \tag{14}$$

Define $U_x = -\sin\theta$ and $U_y = \cos\theta\sin\phi$, thus, the reference angles for the attitude controller could be obtained as

$$\theta_1^d = \arcsin(-U_x) \tag{15}$$

$$\phi_1^d = \arcsin\left(\frac{U_y}{\cos(\theta_1^d)}\right) \tag{16}$$

Notice that the function $\arcsin(\cdot) \in (-1,1)$ this makes a restriction in the attitude control cause small movements have to be considered. U_x and U_y will be obtained from the consensus algorithms.

IV. Multi-agent systems and consensus algorithm

The communication in multi-agent system uses oftenly the *Graphes* theory, which defines a representation showing how the communication flows between nodes (in our case agents). A graph is a set of ordered pairs (V_n, ε_n) where $V_n : 1, n$ are the agents and $\varepsilon_n \subseteq V_n \times V_n$ defines the edges or the communication interaction. In our work, directional graphs are considered, this means that the graph (p,q) denotes that the agent q can obtain information from vehicle p but not *vice versa*. Notice that, directed graph is a general case of the indirected graphs. Usually the edge is considered to other agent, the self-loop edge is not allowed, at least in this paper.

There are matrices, associated with graphs, for describing mathematically the interaction between agents, these matrices are the *Laplacian, Degree* and *Adjacency* matrix. The Degree matrix is diagonal in which its entries are the degree of the node, this is the number of agents that are sending information to the agent. Its mathematical representation is $D = \text{diag}(d_1, ..., d_N)$. The adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ is associated to the graph and is defined as

$$a_{ij} = \begin{cases} a_{ii} = 0 \\ a_{ij} > 0 \quad \text{if}(v_j, v_i) \in V_i \times V_j \end{cases}$$

Finally, the *Laplacian* matrix is defined as

$$L = D - A$$

A directed path from the node V_{i1} to the V_{il} is a defined sequence of interactions of the way $(V_{ik}, V_{i(k+1)})...(V_{i(l-1)}, V_{il})$. A graph is strongly connected if there is at least one direct path from any node to other one in the graph. The previous is one of the conditions to achieve the convergence of the fleet.

A. First Order dynamics

For the consensus algorithm a first order dynamics is used for the analysis, this does not means that the system (in our case the quadrotor) has this dynamic. The dynamic of the information of states is given by

$$\dot{X}_i = u_i, \qquad i = 1,...,n \tag{17}$$

where $X \in \mathbb{R}^m$ is the information state and $u_i \in \mathbb{R}$ is the information control of the i-th agent. The consensus algorithm can be achieved with

$$u_i = -\sum_{j=1}^{n} \Big(a_{ij}(t)(X_i - X_j) \Big) \qquad i = 1,...,n \tag{18}$$

where a_{ij} is the (i,j) entry of the adjacency matrix, the controller in matrix representation is

$$\dot{X} = -[L(t) \otimes I_m]X \tag{19}$$

where $X = \begin{bmatrix} X_1 & \dots & X_n \end{bmatrix}^T$ are the information states, in this case we are interested to achieve the consensus in the xy plane, then, $X_i = (x_i, y_i)$ and it defines the positions of the i-th agent, therefore, the problem becomes a Linear Time Invariant analysis.

Then (18) can be modified adding a relative distance and a speed terms and also introducing the leader information, hence

$$u_i = -\tanh\Big(\sum_{j=1}^{n} (X_i - X_j) - \Delta_{ij} \Big) \tag{20}$$

$$u_{leader} = -\tanh\Big[\Big(\sum_{j=1}^{n} (X_i - X_j) - \Delta_{ij} \Big) - (X_d - X_{leader}) \Big] \tag{21}$$

Function $\tanh(\cdot)$ is used as saturation to produce small movements as desired for obtaining θ_d and ϕ_d. This protocol algorithm can be seen as a proportional controller in which the position error is computed with the positions of all agents in the fleet.

B. Backstepping consensus algorithm

The system (14) will be used for consensus and flight formation, notice that for achieving these ones with the dynamics information of first order is complicate, cause the agents could have oscillations when arriving to the reference even if it is very smooth. Similar results are obtained when using a feedback to transform the second order system into a first order, as suggested by some authors.

For agent's position control and consensus and formation between them, the Backstepping approach is used with some modifications. This methodology allows us to manipulate each state and by consequence the convergence analysis of the fleet. Some ideas from [1] and [3] are taken for the stability analysis.

Rewriting (14)

$$\dot{x}_{1_i} = \dot{x}_{2_i} \tag{22}$$

$$\dot{x}_{2_i} = Ux_i \frac{1}{m_i} U_{1_i} \tag{23}$$

$$\dot{y}_{1_i} = \dot{y}_{2_i} \tag{24}$$

$$\dot{y}_{2_i} = Uy_i \frac{1}{m_i} U_{1_i} \tag{25}$$

where the sub-index i denotes the number of the agent. It is assumed homogeneity in the agents.

Define the tracking error between agents as

$$\hat{e}_{1_i} = -\sum a_{ij}(x_{1_i} - x_{1_j}) \quad \forall j : 1,...,n \tag{26}$$

Note that the $\hat{e}_{1_i}$ can be represented as

$$\hat{e}_{1_i} = -\Big(\sum a_{ij}\Big) x_{1_i} + \sum a_{i_j} x_{1_j} \quad \forall j : 1,...,n \tag{27}$$

then

$$\dot{\hat{e}}_{1_i} = -\Big[\sum a_{ij}\Big] x_{2_i} + \sum a_{ij} \dot{x}_{1_j} \tag{28}$$

A positive function is proposed and its time derivative as

$$V(\hat{e}_{1_i}) = \frac{1}{2}\hat{e}_{1_i}; \qquad \dot{V}(\hat{e}_{1_i}) = \hat{e}_{1_i}\dot{\hat{e}}_{1_i} \tag{29}$$

by substituting (28) into (29)

$$\dot{V}(\hat{e}_{1_i}) = \hat{e}_{1_i}\Big[-\Big[\sum a_{ij}\Big] x_{2_i} + \sum a_{ij}\dot{x}_{1_j} \Big] \tag{30}$$

Define $x^v_{2_i}$ as

$$x^v_{2_i} = \frac{1}{\sum a_{ij}} \Big[\sum a_{ij}\dot{x}_{1_j} + \bar{\alpha}_1 \hat{e}_{1_i} \Big] \qquad \bar{\alpha}_1 > 0 \tag{31}$$

It is clear to note that if $x_{2_i} \to x^v_{2_i}$, then

$$\dot{V}(\hat{e}_{1_i}) = -\bar{\alpha}_1 \hat{e}^2_{1_i} \tag{32}$$

and if $\hat{e}_{1_i} = 0$ then $\dot{V}(\hat{e}_{1_i}) = 0$ implying that the consensus has been achieved, hence the first state is Globally Asymptotically Stable (GAS).

Therefore propose $\hat{e}_2 = x_{2_i} - x^v_{2_i}$ and representing the system in terms of the errors $\hat{e}_{1_i}, \hat{e}_{2_i}$, it yields

$$\dot{\hat{e}}_{1_i} = -\hat{e}_{2_i} - \bar{\alpha}_1 \hat{e}_{1_i} \tag{33}$$

$$\dot{\hat{e}}_{2_i} = U_{x_i} - \frac{1}{\sum a_{ij}} \Big[\sum a_{ij}\ddot{x}_{1_j} + \bar{\alpha}_1 \dot{\hat{e}}_1 \Big] \tag{34}$$

Propose the following Lyapunov function for the new system representation

$$V(\hat{e}_{1_i}, \hat{e}_{2_i}) = \frac{1}{2}\hat{e}^2_{1_i} + \frac{1}{2}\hat{e}^2_{2_i} \tag{35}$$

then

$$\dot{V}(\hat{e}_{1_i}, \hat{e}_{2_i}) = \hat{e}_{1_i}\dot{\hat{e}}_{1_i} + \hat{e}_{2_i}\dot{\hat{e}}_{2_i} \tag{36}$$

Introducing (33) and (34) into (36)

$$\begin{aligned} \dot{V}(\hat{e}_{1_i}, \hat{e}_{2_i}) =& \hat{e}_{1_i}(-e_{2_i} - \bar{\alpha}_1 \hat{e}_{1_i}) + e_{2_i}\Big[U_{x_i} + \\ & \frac{1}{\sum a_{ij}} \sum a_{ij}\ddot{x}_{ij} + \bar{\alpha}_1 \dot{\hat{e}}_{1_i} \Big] \end{aligned} \tag{37}$$

Selecting the control law as

$$U_{x_i} = \sum a_{ij} \sum a_{ij} \ddot{x}_{ij} - \bar{\alpha}_1 \dot{\hat{e}}_{1_i} - \bar{\alpha}_2 \hat{e}_{2_i} + \bar{\alpha}_1 \hat{e}_{1_i} \hat{e}_{2_i} \tag{38}$$

implies that

$$\dot{V}(\hat{e}_{1_i}, \hat{e}_{2_i}) = -\bar{\alpha}_1 \hat{e}_{1_i}^2 - \bar{\alpha}_2 \hat{e}_{2_i}^2 \leq 0$$

hence the system will have a stable behavior. Similar results are obtained when using this methodology for computing the control position for the y axis.

V. Numerical Validation

In this case, the algorithm is done for three agents but it can be extended for n-agents. The nominal parameters for each quadrotor for simulation purposes and the gains used in simulations are

Parameter	Value	Parameter	Value
Ω	0.01	g	9.81 $\frac{m}{s^2}$
$\frac{I_z - I_x}{I_y}$	0.01	m	0.5 kg
$\frac{I_x - I_y}{I_z}$	0.001	$\frac{I_y - I_z}{I_x}$	0.001
$\frac{I_r}{I_y}$	0.001	$\frac{I_r}{I_x}$	0.001
$\frac{I_r}{I_y}$	0.001	$\frac{l}{I_x}$	0.15
$\frac{l}{I_y}$	0.15	$\frac{1}{I_z}$	0.15
α_i	5		

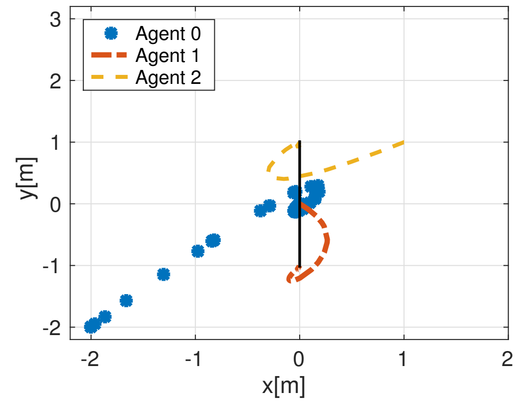
For simulations three formations are considered and two topologies, firstly, the are considered to shape a vertical and horizontal line between quadrotors with a relative distance of 1 meter between consecutive agents and the last formation is a triangular formation. In the first formation, the initial position of each agent is not relevant, in fact it could be in a random position in the xy plane and the last position are considered to make a vertical line between quadrotors as can be seen in Figure 1a.

In Figure 1b the agent 0 tries to keep its position and the agent 1 and 2 are moving to change the formation to a horizontal line, notice that in this simulation the center of the formation is not moved.

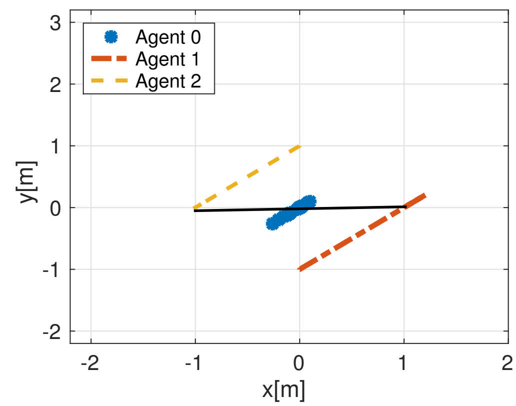
A triangular formation is presented in Figure 1c but not just the formation is shown but also the fleet of drones are moving this is to verify that this algorithm is useful to follow a desired trajectory.

In the following equation the topologies considered for the simulations are shown and they are related with the Laplacian matrices.

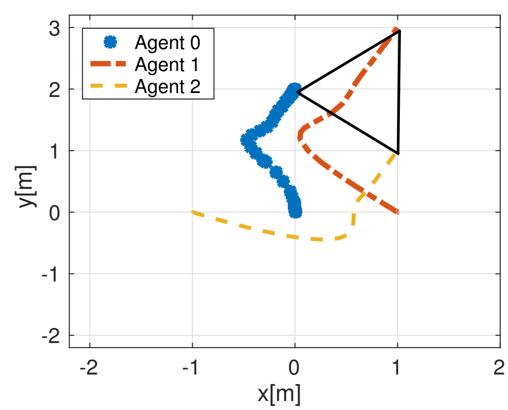
$$L_1 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \qquad L_2 = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \tag{39}$$



(a) Home to horizontal formation



(b) Horizontal to vertical formation



(c) Vertical to triangular formation

The Matlab simulation is not the only numerical validation for the flight formation algorithm, at the Heudiasyc Laboratory a simulator has been developed and is very close to the real validation, in fact, once the algorithm is proved in this simulator the possibilities to achieve the experimental validation in the laboratory platforms are very high, see Figure 2.

For the tests in this simulator, three quadrotors are considered, at the beginning they have default or home positions and can be modified anytime for the user, when the algorithm begins the agents form a horizontal line, the second

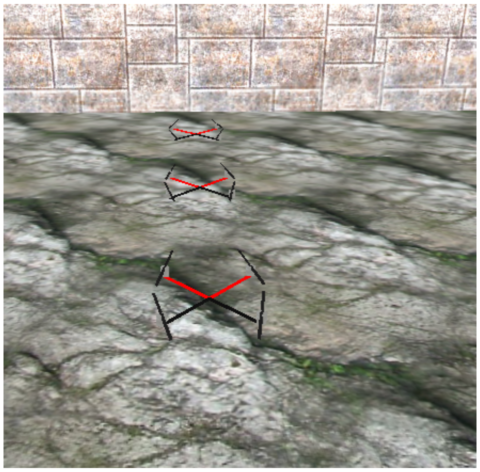


Fig. 2: Flair simulator developed at Heudiasyc laboratory.

configuration is a vertical line and the last one is a triangular, all the formations are done continuously, this is, while the agents are flying they can change the positions to achieve each formation, the behavior of each flight formation can be seen in Figure 4.

The last test is the trajectory tracking while the quadrotors try to keep a triangular formation, in Figure 3 the performance of the algorithm.

VI. Conclusion

A Backstepping algorithm was presented to achieve flight formation between aerial agents, it has been tested to control the position of three agents with quadrotor dynamics. The control strategy was obtained in detail showing the stability analysis via the Lyapunov theory.

The change of communication topology was done while the quadrotors are flying and it has been observed that the performance was not degraded.

Future work includes the real validation in our aerial platforms of the proposed algorithm.

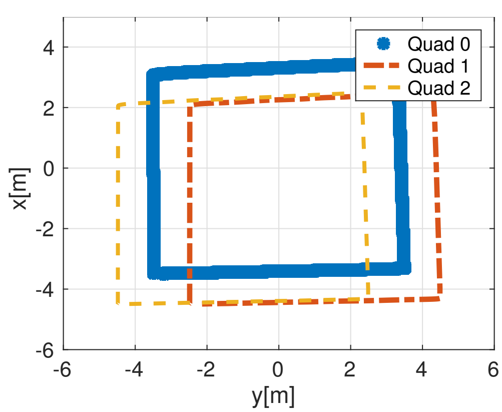
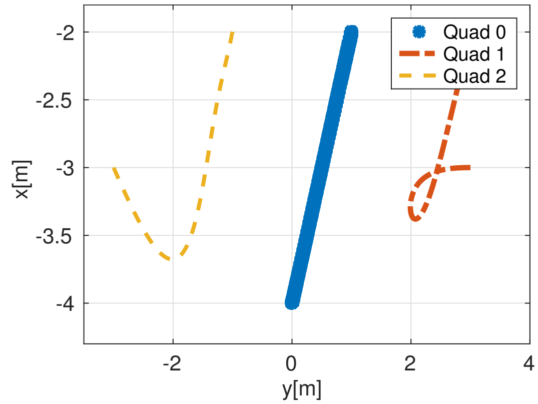
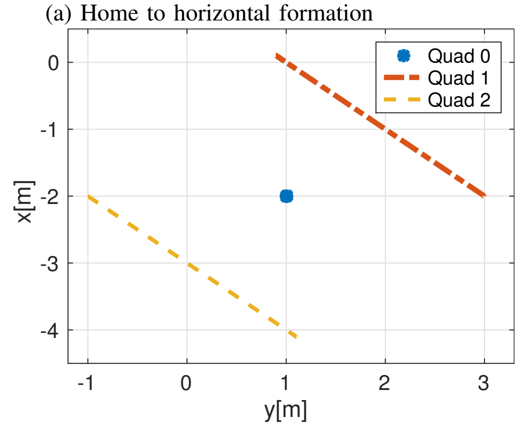


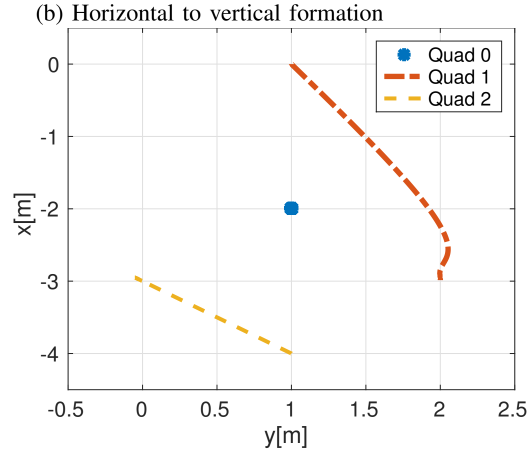
Fig. 3: Flight formation with trajectory tracking.



(a) Home to horizontal formation



(b) Horizontal to vertical formation



(c) Vertical to triangular formation

Fig. 4: Numerical validation with the Flair simulator/

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