About the Dimension of Discrete Sets

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Within the context of Digital Image Analysis, discrete sets (discrete subspaces of R^n), which appear as domains of definition of digital images, commonly are modelled by adjacency graphs (graph theoretic approach) or by cell complexes, or, equivalently, by T_0 Alexandroff topological spaces (topological approach). Both approaches provide the possibility to define and to study topological properties of digital objects (subsets of discrete sets).

Dimension is an important topological characteristic, also for digital images; it is essential for defining digital curves and surfaces. Based on the model of T_0 Alexandroff spaces, in previous works of the author, a topological dimension function was proposed for (the domain of) digital images: the small inductive dimension of General Topology, adapted to Alexandroff spaces, which, for T_0 spaces, turns out to coincide with the poset dimension (given as the supremum over the lengths of chains of the corresponding specialization order) (1993). Conditions for the preservation of dimension under digitization were studied (2002/2004), and this dimension was considered in the analysis of skeletons (2008).

Another dimension was proposed by Mylopoulus and Pavlidis (1971), based on modelling a discrete set as a finitely presented Abelian group (with finitely many generators and relations). This dimension was proved to be equal to the number of generators, for free Abelian groups (which has no other relations than commutativity and inversion). Adjacency graphs used for modelling digital images, can be interpreted as such Abelian groups (for example, the 4-adjacency on Z^2 is a free group, the 8-adjacency is not free). Although, at its origin, there is no topology present in the structure under consideration, the authors named their dimension a topological dimension, and some properties similar to properties of topological dimension functions were proved. Since 2004, the Mylopoulus-dimension has been popularized as a topological dimension for digital images defined on Z^n and modelled by adjacencies, by various authors. In particular, for the case Z^2, this dimension was used to define digital curves and surfaces.

Despite of the fact that the Mylopoulus-dimension and the small inductive dimension are defined on completely distinct mathematical structures, both dimensions describe “topological” properties of the same digital object. In our talk, we perform an analysis of the Mylopoulus-dimension, and of some properties of our topological dimension, and we construct a platform to make possible the comparison of both dimensions.

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