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Thinning on posets related to polygonal tilings and to digital images

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Thinning is a procedure widely used within Digital Image Processing and Analysis, in order to reduce a digital object to a subset named skeleton. Skeletons are important tools for the analysis, description and recognition of digital images. Intuitively, the skeleton of an object is an "irreducible" subobject which "represents the topology of the object in the homotopy sense". Thinning is an iterative procedure where in each iteration, certain ("simple") elements of the frontier of the remaining object are deleted in such a manner that the preservation of connectivity (of objects and of background) as well as of "thin and long" parts of the object, are guaranteed.

Formally, the skeleton of an open proper subset M of R^n is the set of all points p of M with the property that there exist two distinct points a, b on the frontier of M such that $d(a,p)=d(b,p)$ is equal to the Euclidean distance of p to the frontier of M (Blum 1967). Each skeleton point is the centre of a maximal open ball contained in M. The literal application of Blum's definition to a digital image, defined on a discrete set which commonly is modeled by some adjacency graph, generally produces skeletons which are not connected or not "thin", or not situated on the "central line", applying any metric. For this reason, there is no generally accepted definition of a digital skeleton so far; very commonly the skeleton is defined to be the result of certain algorithm applied to the object, for example, of thinning.

Our talk deals with skeletons in digital objects defined on discrete sets modeled by abstract cell complexes as considered by Kovalevsky (since 1989), constructed from polygonal tessellations of the Euclidean plane. Such complexes are posets, and, for a suitable dimension function, there are equivalent to certain topological T_0 Alexandroff spaces which were introduced as "digital spaces" by Kronheimer (1989). We define the skeleton of an object modeled by a finite subcomplex to be the result of a thinning algorithm published by Kovalevsky (2001), which alternately eliminates simple non-end elements from the (topological) frontier and from the open frontier of the object.

In our talk, we give a theoretical foundation of the algorithm: we report about our results about the relation between global and local simplicity, and that the parallel implementation of Kovalevsky's algorithm preserves topology. For our proofs, we apply a digital Jordan theorem due to Neumann-Lara/Wilson (1992) which is valid in any planar locally Hamiltonian graph. It is of independent interest that the incidence graph of the cell complex constructed from any polygonal tiling has these properties, fact we also proved.

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